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GYROMAGNETIC RATIOS FOR FERROMAGNETIC SUBSTANCES: NEW DETERMINATIONS AND A NEW DISCUSSION OF EARLIER DETERMINATIONS

By S. J. Barnett

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Part I

INTRODUCTION AND REFERENCES

§ 1. Introduction. This paper is a sequel to three others published in these Proceedings in and before 1934, and referred to below as II, III and V. It is concerned chiefly with the latest stage of a long investigation which has passed through five distinct earlier stages as follows:

(1). 1914. The discovery of magnetization by rotation, or the Barnett effect, by a method of electromagnetic induction, and the measurement, with an error of only a few per cent, of the gyromagnetic ratio for iron, showing that it is approximately equal to m/e for a negative electron in slow translatory motion, and thus identifying Ampère's whirls as (at least chiefly) spinning negative electrons.

(2). 1915. The confirmation and refinement of the results of 1914 by means of improvements in the same method.

(3). 1917. Rough measurements by a magnetometer method showing that iron, nickel and

cobalt all have gyromagnetic ratios of the same sign and of nearly the same magnitude.

(4). 1920–1925. A very elaborate and much more precise investigation, by magnetometer methods, of the same effect in many ferromagnetic substances, showing conclusively that the mean gyromagnetic ratio is several per cent greater than m/e and *apparently* differs for different ferromagnetic materials. The results thus indicated that while the spinning electron is the principal magnetic element the orbit also participates to a greater or less extent.

(5). 1927–1934. A long series of precise measurements of the gyromagnetic ratios of iron, permalloy, nickel, and cobalt by means of the converse effect, rotation by magnetization, or the Einstein and de Haas effect. These experiments confirmed in general the results of (4) on the same materials, increased the precision, and showed conclusively that the gyromagnetic ratio varies from material to material.

The objects of the present paper are as follows:

(1) To present an account of new measurements of the gyromagnetic ratios of many ferromagnetic substances by means of the Einstein and de Haas effect, or rotation by magnetization. For a number of the substances investigated this effect has not hitherto been studied, and it has been considered important to extend the work already done on the others. The paper has been somewhat slow in following its predecessors because the new work has required a vast amount of labor both on my own part and on the part of those who have assisted me.

(2) To present, as a result of such measurements, more precise evidence on the question of the location of the angular momentum produced in experiments of this kind.

(3) To revise slightly, in view of the new experiments on e/m and other work, the results obtained by L. J. H. Barnett and myself in our Washington work on magnetization by rotation.

(4) To present some hitherto unpublished, and comparatively rough, results in this field obtained by visitors to the laboratory, partly on account of their historical interest and partly because they show the possibilities of this method even in the daytime, under very adverse conditions.

(5) To compare and discuss so far as necessary the results obtained by the two different methods of research and by different investigators.

§ 2. References to Earlier Papers. It will be conducive to brevity in this paper to refer to the more detailed articles in this field which have hitherto emanated from the author's laboratories by the numerals which are affixed to the titles in the following list:

I. Magnetization by rotation. Phys. Review **6**: 239–270, 1915.

II. New researches on the magnetization of ferromagnetic substances by rotation and the nature of the elementary magnet. (With L. J. H. Barnett.) Proc. Amer. Acad. **60**: 127–216, 1925.

III. Researches on the rotation of permalloy and soft iron by magnetization and the nature of the elementary magnet. Proc. Amer. Acad. **66**: 273–348, 1931.

IV. A new electron inertia effect and the determination of m/e for the free electrons in copper. Phil. Mag. **12**: 349–360, 1931.

V. The rotation of cobalt and nickel by magnetization and the gyromagnetic ratios of their magnetic elements. Proc. Amer. Acad. **69**: 119–135, 1934.

VI. Gyromagnetic and electron-inertia effects. Reviews Modern Phys. **7**: 129–166, 1935.

Part II. WORK ON ROTATION BY MAGNETIZATION

A. General Experimental Methods and Examples

§ 3. Experimental Methods, Apparatus, and Errors in the New Work. All the work undertaken since that published in V has been done by three of the general resonance methods described in III, V, and VI. Reference must be made to these papers for most of the experimental details and for discussions of most of the numerous errors which have to be avoided or eliminated. Modifications adopted in the methods and in the apparatus, and some additional sources of error, will be discussed in appropriate places below.

In most of the work described in III and V the magnetizing coil was wound rigidly on the rotor. In much, perhaps most, of the new work, the magnetizing coil has been fixed to the suspension supports. A diagram of the chief experimental circuits is given for this case in Fig. 1. C is the rotor proper, D the magnet-mirror holder, E the mirrors, F the permanent magnets. B and A are the fixed magnetizing and induction solenoids, and B'–A' their mutual inductance compensator. G and H are the torque and quadrature coils, and G'–H' their mutual inductance compensator. W is the commutator, U the asymmetry reversing switch, R the torque coil reversing switch, P and

S the quadrature coil switches, L and M the quadrature coil voltage dividers (about 18 ohms each) for the two positions of U and P, which are operated together, and QQ' the coils of high resistance in the quadrature coil circuit. The earth connections at Z and Y were not always made, neither was the connection between IJ and G' always made.

§ 4. METHOD I. Most of the observations have been made by the *Large deflection method II* of III § 26, which is identical with the *Large deflection method (a)* of VI §30, with the modifications indicated in the next paragraph (§5). In this method the conductance $X_0 + \delta X_0$ of the secondary circuit at which the resonance amplitude is a minimum is determined by measuring *accurately* the amplitudes A and A_2 (= approximately A), respectively, for $X = 0$ and $X = X_2$ = approximately $2(X_0 + \delta X_0)$; by measuring *approximately* the amplitude A_0, the approximate minimum amplitude, for X = approximately $X_0 + \delta X_0$; and by substituting in the formula

$$X_0 + \delta X_0 = \frac{(A^2 - A_0^2)^{1/2}}{(A^2 - A_0^2)^{1/2} + (A_2^2 - A_0^2)^{1/2}} \quad (4\text{-}1)$$

X_0 is the conductance at which the amplitude would vanish if there were no extraneous torques; and δX_0, the error due to such torques, is eliminated by making observations for both directions, I and II, of the asymmetry reversing switch U in the magnetizing circuit, and for two azimuths, NPE and NPW, differing by 180°, of the rotor and permanent magnet attached to it. The mean value of δX_0 for these four arrangements is zero, and the gyromagnetic ratio is determined from the equation

$$-\rho = X_0 \Gamma_0 \gamma_0 m_0 \quad (4\text{-}2)$$

where Γ_0 and γ_0 are the constants of the *torque coil* and *induction solenoid*, respectively, and m_0 is the moment of the permanent magnet. If the magnetizing coil is wound upon the rotor, small corrections must be applied for electron-inertia in the coil and for its magnetic moment.

In a comparatively small part of the new work the method was slightly modified in the manner described at the end of V §2, p. 123. In this modification $X_0 + \delta X_0$ was obtained by taking one-half the value of X for which the precisely measured amplitude (beyond the minimum) was *equal* to the amplitude (also measured with precision) for which $X = 0$. The first value was, as in V, not observed directly, but was obtained by interpolation or extrapolation for values of X

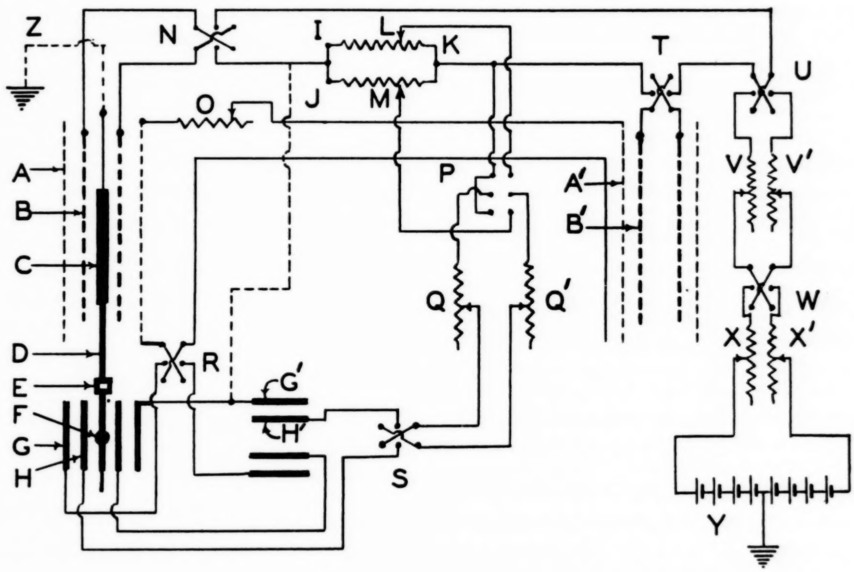

FIGURE 1. Diagram of the principal apparatus as arranged for work with the magnetizing coil fixed to the earth.

near $2(X_0 + \delta X_0)$; and $X_0 - \delta X_0$ was obtained in the same way. Repeated observations were made as in the unmodified method, an example of which is given in §5.

§ 5. Example of Method I. An example of Method I, as modified for nearly all of the work described here, is given in Table 1, which contains the amplitude observations on the long wound 2 mm. permalloy rotor made on January 14–15, 1935, and the results. The vibration observations began at 11:31 p. m. and ended at 2:50 a. m. The intensity of the magnetizing field was about 35 gausses, and the frequency 9.8 per second. The vertical intensity of the earth's field was doubled, instead of being annulled, as usually. The moment of the rotor was about 640 e.m.u. and that of its coil (traversed by a current of 0.1 ampere) about 4.5 e.m.u. The moment of the permanent magnet was 0.684 e.m.u.

Instead of making a single measurement of amplitude for each setting of the resistance and reversing switch, the measurements (in all methods used) have long been made in groups of 3, with an interval of half a minute between the first and second and second and third. This enables the observer to be certain that sufficient time has been allowed to elapse between the different settings. For each of the azimuths (1) and (2) two groups A and B of amplitude determinations are made, each group containing 24 separate measurements taken, as in the earlier work, on a strict time schedule. The numbers in parentheses under A and B give the order of the observations. Between (3) and (4), (9) and (10), (15) and (16), and (21) and (22), there is an interval of 2^m; between (6) and (7), (18) and (19), an interval of 3^m. Between the other successive numbers there is an interval of $\frac{1}{2}^m$. Except the last interval, which has almost always been kept the same, the intervals are not the same for all rotors, as they have very different time constants. For each of the azimuths (1) and (2), or north pole of the vibrator magnet east and north pole west, and each position of the reversing switch, I and II, the values of $\rho e/m$ are given, together with the final means, at the foot of the table. The result, 1.057, like the other values for this rotor in Series IV, Table 40–1, is greater than the mean for the other series, possibly because of the relatively intense vertical magnetic field in which the vibrations were made.

§ 6. Method II. This is the *Large deflection method I* of III § 20, and the *Large deflection method (c)* of VI § 30, B, with improvements. It was but little used in III, and was there incompletely described and discussed. In it a comparison is made between (1) the amplitude A_G of the vibration produced by the first harmonic of the gyromagnetic torque $g = G \cos \omega t$ when there is no current in the torque coil, and (2) the amplitude A_c of the vibration produced by the first harmonic of an independent current $c = C \cos (\omega t + \delta)$ in the torque coil when there is no gyromagnetic torque acting. The magnitude of the gyromagnetic ratio is given by the equation

$$|\rho| = (\Gamma_0 m_0 C/\omega\mu_0)\, A_G/A_c \qquad (6\text{-}1)$$

where μ_0 is the amplitude of the first harmonic of the magnetic moment of the magnetic material in the rotor.

A considerable amount of time has been spent in developing this method, but in some of the work the precautions taken or the quadrature coil data (see § 32) have not been such as to justify the inclusion of the results here.

In practice the same reversals made in the first method are required to eliminate extraneous torques in phase with or in opposition to the gyromagnetic torque; also separate experiments must be made to test for and measure any quadrature torque present. If there is present a quadrature torque which alone would produce an amplitude A_0, the amplitude A_G in (1) would be found from the *measured* amplitude A by means of the relation $A_G = (A^2 - A_0^2)^{1/2}$.

If the magnetizing coil is wound rigidly on the magnetic material and has a moment whose amplitude is μ_0', it will give rise to a torque with amplitude $\rho'\omega\mu_0'$ which is in phase with the gyromagnetic torque with amplitude $\rho\omega\mu_0$ and alone would produce an amplitude A_E. Here ρ' is the ratio of the angular momentum of an electron moving in a circle to its magnetic moment, viz. $2\, m/e$ (see IV). The observed amplitude in case (1), corrected for any quadrature effect, will now be $A = A_G + A_E$, and we shall have

$$\frac{A(\equiv A_G + A_E)}{A_c} = \frac{\rho\omega\mu_0 + \rho'\omega\mu_0}{\Gamma_0 m_0 C}$$

or

$$\rho = \frac{\Gamma_0 m_0 C\, A}{\omega\mu_0\, A_c} \cdot \frac{1}{1 + \rho'\mu_0'/\rho\mu_0} \qquad (6\text{-}2)$$

Since $\rho'/\rho \lessdot 2$, and $\mu_0'/\mu_0 << 1$, this equation becomes, with a sufficient degree of precision,

$$|\rho| = \frac{\Gamma_0 m_0 C\, A}{\omega\mu_0\, A_c}(1 - \rho'\mu_0'/\rho\mu_0) \qquad (6\text{-}3)$$

TABLE 1

OBSERVATIONS ON 2 MM. WOUND PERMALLOY ROTOR, JANUARY 15, 1935, AND RESULTS

I. 2 × amplitude in cm.

(1) NORTH POLE EAST

Mean $2A_0$ I = 0.50† Mean $2A_0$ II = 0.20†

A

∞ I			2168.2 II		Resistance*	∞ II			2168.2 I		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
4.86	4.86	4.84	4.80	4.82	4.80	4.62	4.62	4.62	4.76	4.74	4.78
(24)	(23)	(22)	(21)	(20)	(19)	(18)	(17)	(16)	(15)	(14)	(13)
4.88	4.92	4.90	4.82	4.82	4.86	4.62	4.61	4.62	4.74	4.72	4.76

B

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
4.92	4.90	4.90	4.92	4.94	4.92	4.62	4.62	4.60	4.86	4.86	4.84
(24)	(23)	(22)	(21)	(20)	(19)	(18)	(17)	(16)	(15)	(14)	(13)
4.86	4.86	4.90	4.90	4.88	4.90	4.64	4.64	4.62	4.82	4.84	4.82
Means 4.883			4.865			4.621			4.795		

(2) NORTH POLE WEST

Mean $2A_0$ I = 0.20 Mean $2A_0$ II = 0.65

A

∞ I			2238.2 II		Resistance*	∞ II			2168.2 I		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
4.80	4.82	4.80	4.84	4.82	4.86	4.70	4.70	4.71	4.72	4.74	4.74
(24)	(23)	(22)	(21)	(20)	(19)	(18)	(17)	(16)	(15)	(14)	(13)
4.78	4.80	4.80	4.78	4.80	4.80	4.68	4.68	4.70	4.76	4.72	4.70

B

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
4.86	4.84	4.84	4.80	4.82	4.86	4.74	4.76	4.72	4.76	4.78	4.78
(24)	(23)	(22)	(21)	(20)	(19)	(18)	(17)	(16)	(15)	(14)	(13)
4.78	4.80	4.80	4.76	4.80	4.82	4.68	4.70	4.70	4.76	4.82	4.80
Means 4.810			4.813			4.706			4.757		

II. $\rho \times e/m$

NPE I	NPE II	NPW I	NPW II
1.076	1.038	1.083	1.030

Mean NPE = 1.057 Mean NPW = 1.057
Mean I = 1.080 Mean II = 1.034
Final mean (I and II or NPW and NPE) = 1.057

* This is the resistance in the induction circuit in ohms.

† A_0 I and A_0 II are the minimum amplitudes for the positions I and II of the half-cycle asymmetry switch.

In the practice of the method it is important that the natural frequency of the system be identical in (1) and (2), and thus that the heating effect of the magnetizing coil be the same in the two cases. For this reason the magnetizing coil in (2) is traversed by a current of the same magnitude as in (1), but from an independent source with a frequency so remote from that of the vibrating system, or any of its low odd harmonics, that it produces no appreciable amplitude.

If the waves of c and μ are *strictly rectangular*, whether the gap width is small or large, and if the contact resistance between brushes and commutator segments is independent of the motion, the ratio of C to μ_0 in (6-1, 2, 3) is equal to the ratio of the steady current in (2) to the value of the magnetic moment of the rotor for the steady value of the magnetizing current in (1), *while the commutator is at rest.*

As a matter of fact the waves are, of course, not strictly rectangular, and their shapes differ on account of the difference between the time constants, the difference between the sparks at the commutator, and hysteresis. With low frequencies the assumption of true rectangular waves is much nearer the truth than with high frequencies, so that lower frequencies will, other things being equal, give better results than higher.

In the method as finally practised, the current C was produced by a voltaic cell in series with a high resistance box, and the steady value of the current for (2) was determined as the ratio of the electromotive force to the total resistance of the circuit; while in (1) the mean magnitude of the commutated magnetizing current was maintained steady as read accurately from an ammeter on the battery side of the commutator. In separate experiments the ratio of the steady current with the commutator at rest to this mean commutated value was repeatedly determined.

Moreover, from the fact that the time constant in (1) is greater than that in (2), it is easy to show that even if the gap correction were zero the apparent value of ρ would be too large, the inequality being greater the greater the time constant in (1), and the greater the frequency. Errors arising from these causes, together with the necessity of measuring accurately several quantities not needed in the determination of ρ by the other methods, are the chief disadvantages of Method II.

While the method clearly has disadvantages as compared with Method I, it has one advantage in that no secondary circuit is used, so that there is no chance of a leak between the two circuits introducing an extraneous torque with its attendant possible systematic error. By paying proper attention to insulation this torque can always be made small; and if it is in phase with the applied electromotive force as assumed in the earlier papers, it will be in quadrature with the gyromagnetic torque and without effect on the results. In this work great attention has always been paid to the insulation between the two circuits, and the measured leakage through the secondary with a galvanometer in series has always been very minute in comparison with the current due to the changing magnetic moment of the rotor. Nevertheless some small discrepancies between results may possibly be accounted for by leakage on the assumption that the actual leakage current immediately after reversal of the commutator is much greater than the steady or nearly steady current measured quickly thereafter, and thus produces a phase shift of its first harmonic from quadrature with the gyromagnetic torque. Polarization effects in dielectric conduction are of course well known. There is thus an advantage in having available a method free from the possibility of such effects.

§ 7. Example of Method II. Table 2 contains the amplitudes obtained in a single set of observations on the 3.2 mm. cobalt-iron rotor, together with the resulting values of $\rho\, e/m$. A mean commutated current of 330.5 milliamperes traversed the fixed coil. The frequency was 4.54 cycles per second. The moment of the rotor (for the maximum value of the current commutated) was 1256 e.m.u. (ratio of maximum to mean current estimated as 361/350 from observations made later).* The moment of the permanent magnet m_0 was 0.746 e.m.u. Mirror holder No. 4 was used. The resistance of the torque coil circuit during the torque coil drive was 60,315 ohms, the e.m.f. of the cell 1.578 volt. Observations began at 3 a. m. and ended at 6:30 a. m.

For each of the azimuths NPE and NPW four groups of amplitude measurements were made, two of them, viz., A and C, with gyromagnetic drive, and two, viz., B and D, with torque coil drive. The order of the groups is A, B, C, D; and the order of observations within the groups is given by the numbers in parentheses. Each group contains 12 measurements of amplitude,

* In later work this ratio was determined separately for each set, an improvement in the procedure being thus effected.

half of them for each position (I or II) of the reversing switch. The interval between (3) and (4), and between (9) and (10) is 3 m. That between other successive observations within the groups is ½ m. The interval between successive groups is about 5 m. The values of $\rho\, e/m$ for the two azimuths and the two positions of the switch, together with the final means are given at the foot of the table, under II.

§ 8. Method III. This is the single straight line form of the graphical method of III § 46 and VI § 30A, with the strict time schedule, the repetition of observation of each kind at half-minute intervals, and the reversals for asymmetry (as well as reversals in azimuth) as practised in the use of the other methods. The amplitude is observed and plotted as a function of the conductance X of the induction circuit for each azimuth and each direction of the asymmetry switch; and for each arrangement $X_0 + \delta X_0$ is obtained from the intersection with the axis of X of the symmetrical straight line best representing the observations and drawn across the axis X. The schedule is so arranged that for each azimuth the mean times are identical for all the values of the conductance and both directions of the asymmetry switch. The mean value of $X_0 + \delta X_0$ is the value X_0 sought, from which ρ is calculated as in Method I.

In the new work only a few observations have been made by this method.

TABLE 2

Observations on 3.2 mm. Cobalt-Iron Rotor, May 28, 1936, and Results

I. 2 × Amplitudes in cm. ($2A_\infty$)

$2A_0$ I = 0.11						NPE					$2A_0$ II = 0.11
A						B					
G I			G II			T I			T II		
(1) 4.70	(2) 4.68	(3) 4.70	(4) 4.60	(5) 4.59	(6) 4.60	(1) 4.62	(2) 4.62	(3) 4.62	(4) 4.60	(5) 4.60	(6) 4.60
(12) 4.70	(11) 4.70	(10) 4.70	(9) 4.62	(8) 4.62	(7) 4.62	(12) 4.60	(11) 4.60	(10) 4.60	(9) 4.60	(8) 4.60	(7) 4.60
C						D					
(1) 4.70	(2) 4.70	(3) 4.70	(4) 4.60	(5) 4.60	(6) 4.50	(1) 4.60	(2) 4.61	(3) 4.61	(4) 4.57	(5) 4.58	(6) 4.58
(12) 4.70	(11) 4.69	(10) 4.70	(9) 4.60	(8) 4.60	(7) 4.60	(12) 4.54	(11) 4.54	(10) 4.55	(9) 4.57	(8) 4.56	(7) 4.57
Mean G_{I} 4.697			Mean G_{II} 4.603			Mean T_{I} 4.592			Mean T_{II} 4.587		
$2A_0$ I = 0.08			Mean $2A_0$ I = 0.095			Mean $2A_0$ II = 0.140			$2A_0$ II = 0.17		
						NPW					
Mean G_{I} = 4.677			Mean G_{II} = 4.652			Mean T_{I} = 4.630			Mean T_{II} = 4.623		
Mean $2A_0$ I = 0.115											Mean $2A_0$ II = 0.040

II. $\rho e/m$

NPE			NPW		
I = 1.061	II = 1.040	Mean = 1.050	I = 1.048	II = 1.044	Mean = 1.046
Mean of all = 1.048					

B. Apparatus: Construction and Constants

§ 9. Rotor and Suspension Construction. Many of the rotors used in this later work have been constructed on the anti-magnetostriction principle used in building the complex cobalt, nickel, and iron rotors described in IV and VI.

As in the case of the standard short rotors of the earlier work, designed to be used with the fixed magnetizing coil only, or with both fixed and moving coils, the magnetic material in most of them has been about 8 cm. shorter than in the case of the standard long rotors. Other rotors have been constructed like the earlier standard short rotors, some with and some without winding. Every rotor has the same total length.

In the most recent complex rotors the terminal construction differs from that described in V, the end of the rotor being turned down to a small diameter, and fitting, with small clearance, in a central cylindrical hole machined in the cap. Also, in some cases, the central tube between the magnetic rod and the brass sheath, which is thoroughly soldered to both, has been increased in length to about 1.5 cm. to assist in keeping the rod and sheath coaxial without strain and with minimum friction at the bearings.

In much of the work the rotors have been connected with the mirror-holder, and this with the lower suspension, by rectangular hooks and rings such as were always used in the earlier work.* In the case of the more recent rotors, and that of earlier rotors recently modified, the ends of the rod (or brass tube) have been provided with threaded brass caps by means of which they can be rigidly screwed to a brass terminal at the bottom of an upper suspension and either to the top of one of the mirror holders of newer design, described below, or to a finely constructed universal joint which is screwed to the bottom of the rotor and to the top of the mirror holder. Flanges at the junctions assist in making the alignment exact and rigid. At least one of the flanges in contact is always slightly recessed, so as to make contact only along a rim at the outer edge.

All of the latest experiments have been made with the fixed magnetizing coil. This fact has made it unnecessary to provide a separate upper suspension and torsion head for each rotor. Three new upper suspensions have been constructed of No. 34, No. 32 and No. 28 German silver wire. Each is about 7 cm. long and is provided with an axially symmetrical brass terminal at the bottom. This terminal ends in a flat surface with a threaded hole in the center. Into this, with flange (or flange rim) against flange, is screwed the brass cap at the top of the rotor. New lower suspensions are similar, except that they are provided with rectangular hooks for attachment to the stretching weight.

The practical advantages of making experiments with unwound rotors are that the difficulty of straightening such rotors is very much less than that of straightening wound rotors, and that with the former it is unnecessary to use individual suspensions. The winding is also difficult and expensive; and the insulation of the windings, in spite of the greatest care and skill on the part of the mechanician, has often broken down and had to be replaced.

In all the recently constructed rotors provided with their individual upper suspensions, and in all of the new suspension caps, both upper and lower, the German silver wire has been anchored in place as described in IV, p. 134, footnote, and as illustrated in IV, Figure 5. In all recently constructed upper suspensions the wire, carefully centered and soldered in the lower portion of the torsion rod, has been brought out above through an oblique hole in the side, wrapped around the rod in a spiral, and soldered in place. No suspension anchored in this way has so far pulled out.

§ 10. The Magnet-Mirror Holders. The holder known as No. 3 (III p. 312) continued in use into the fall of 1935, with no deliberate change except that the gilding of the mirrors, which had become defective, was replaced by films of aluminum-magnesium alloy. This holder, however, made of pieces screwed and soldered together, proved insufficiently rigid. It was therefore replaced by No. 4.

The chief part of No. 4 was made from a continuous phosphor-bronze rod, slightly thicker than the brass rod of No. 3, of nearly the same length, viz., about 33 cm., and with a bronze hook and a square bronze ring* screwed and soldered into the ends, much as in No. 3. On the same side with the open space in the hook, as a counterpoise, was a bronze projection soldered near it to the rod. Bronze pieces to hold the mirrors and magnets were soldered to the rod and milled until their faces were parallel to one another and each

* To facilitate the connection of the mirror-holder to the rotor, when the amber bobbin was in use, the rings at the lower ends of some of the newer rotors used as above have had their openings extended in the vertical direction, instead of being made square, as formerly. This was made necessary by the design of the new torque-quadrature-coil system.

* The ring at the bottom of the holder, into which a hook at the top of the lower suspension was fitted, was later removed and the end threaded. A small brass cap was then threaded to screw on to this end, and was soldered axially to the upper end of the German silver wire of a lower suspension.

pair nearly symmetrical with the axis of the rod. Four small projections, equal in height, were left on each face of the upper pair, and the mirrors attached to them, at first with shellac. Six equidistant grooves were milled in each face of the lower pair to fit magnets of the same kind used before. Two pieces of optically flat glass 6 x 7 mm.[2] in area and 1 mm. thick were obtained for the mirrors, and coated with aluminum.

Three additional holders, Nos. 5, 6 and 7, shorter, lighter, and simpler, were constructed. The principal part of No. 5 was made of a cylindrical rod of bronze cut from the same stock used in No. 4. Parallel and symmetrical flat surfaces were formed on opposite sides of the rod by a milling machine to hold two optically flat mirrors 1 x 4 x 6 mm. in dimensions; and two sets of 6 grooves parallel to these surfaces were milled for 2 x 6 magnets, similar to those used in the cases of No. 3 and No. 4, and similarly arranged. A rectangular hook of bronze, provided with a counterpoise, was screwed and soldered on to the upper end, and a suspension cap similar to that described in connection with No. 4 was screwed and soldered on to the lower end. The complete length of the holder was about 11 cm. The mirrors, covered with aluminum-magnesium alloy, were cemented on with asphalt, the longer sides being horizontal.

No. 6 was No. 5 modified as follows. The ends were cut off, and axially symmetrical brass caps were screwed and soldered on. The caps were centrally threaded and provided with flanges for accurate and rigid connection to the new type of rotor and lower suspension described above. The upper flange, like that at the bottom of the rotor, was about 7 mm. in diameter; the lower about 5 mm.

No. 6 turned out to be quite imperfect, the surfaces of the flanges not being quite parallel. As soon as possible it was therefore replaced by No. 7, which was practically perfect.

Holder No. 7 had nearly the same dimensions and appearance as No. 6. It is illustrated in Fig. 2 which is drawn nearly to scale. The mirrors and magnets were similar to those used in No. 6, and were put on in nearly the same way. The rest of the holder was turned from a single Tobin bronze rod, except for the screw at the lower end, which was made separately for mechanical reasons.

Nos. 5–7 were constructed in order to make the whole vibrating system shorter and more rigid, with the hope of reducing any error due to a possible difference between the effects of driving the rotor with the torque coil and with the magnetizing coil. The subject will be discussed further in § 33.

A diagram showing the relative lengths of the different holders and the locations of the magnets and mirrors is given in Fig. 3.

The magnets of No. 4 were at first put on with shellac, as formerly. Magnets put on it this way cannot be properly heat-treated to maintain their moments constant, a fact which has cost us a large amount of labor. Afterward they were removed and put on with the diamagnetic cement known as "insolute," as were also those of No. 5. The insolute, which is very brittle, was finally replaced by uncolored glyptal, which is both diamagnetic and exceedingly tough. This was used with Nos. 6 and 7 also. After being cemented in place with insolute or glyptal, the magnets were heated at the temperature of boiling water for a day and a night, after which the systems appeared more stable than those not subjected to heat treatment.

Holder No. 4 was later further modified, terminal flanges and screws similar to those of Nos. 5 and 6 being made to replace the hooks. In this state it was designated No. 8.

In all recent work the mirrors, as suggested by Mr. Julius Pearson, have been attached with pitch, which, in case it distorts the mirrors initially, yields in such a way that the distortion soon disappears. It has produced no appreciable

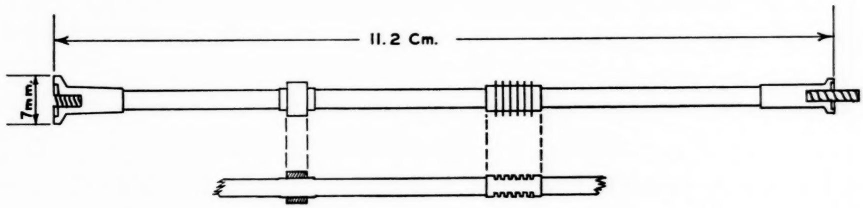

FIGURE 2. Magnet-mirror holder no. 7.

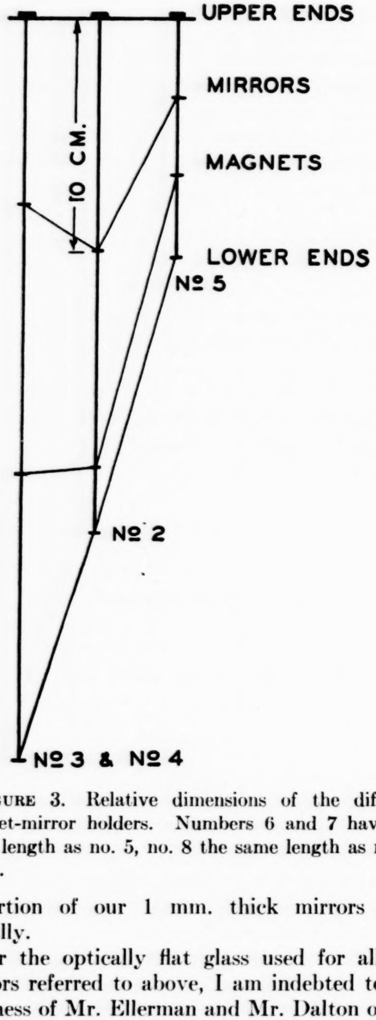

Figure 3. Relative dimensions of the different magnet-mirror holders. Numbers 6 and 7 have the same length as no. 5, no. 8 the same length as nos. 3 and 4.

distortion of our 1 mm. thick mirrors even initially.

For the optically flat glass used for all the mirrors referred to above, I am indebted to the kindness of Mr. Ellerman and Mr. Dalton of the Mt. Wilson Observatory; and for coating these mirrors, as well as other mirrors, with beautiful films of aluminum and aluminum-magnesium alloy, I am indebted to the kindness of my colleagues, Prof. John Strong and Prof. H. W. Edwards.

§ 11. Torque Coils and Quadrature Coils. In the interest of better insulation the twin-wound torque and quadrature coils *B*, and their facsimile compensators, were replaced by coils of nearly the same dimensions, but wound in separate channels in ebonite side by side. The outer pair of one combination constituted the torque coil *C*, the inner, the quadrature coil *C*. The coils were impregnated and covered with white ceresin wax to protect them and to improve the insulation. This was at first very high but soon greatly decreased. There are two varieties of white ceresin wax, one of which, the "natural" variety, appears greatly to surpass the other in surface insulation in moist atmospheres. On the suspicion that the inferior variety had been used, the coils were removed from the bobbins, which were cleaned, and then rewound to nearly the same dimensions. They were then impregnated with "natural" ceresin wax. This gave us torque coil *D*, with its quadrature coil, and the compensators. The constants of the new torque coils, and one of the quadrature coils, were determined electrically with precision by the methods used with *A* and *B*. The constants of torque and quadrature coils *C* were found to be 1129.8 and

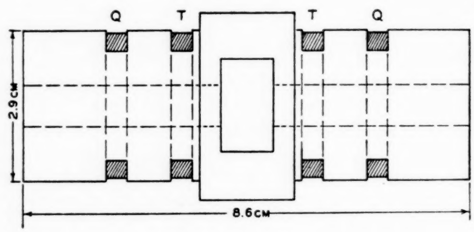

Figure 4. Torque coil (TT) and quadrature coil (QQ), system E. Horizontal section through axis.

2132.1 e.m.u. respectively. The constant of the torque coil *D* was found to be 1083.8 e.m.u.

In order to increase the insulation still further and at the same time to provide coils with constants still more nearly unalterable with time, and with smaller magnetic moments for use with the short magnet-mirror holders Nos. 5–8, another set of torque and quadrature coils of improved design, *E*, was constructed with coils of smaller diameter and a bobbin entirely of amber, in one piece. The coils were covered with natural ceresin wax. A duplicate was constructed to serve as mutual inductance compensator. The details of construction are given in Fig. 4. The inner two coils were used in series as torque coil, the outer two in series as quadrature coil.

The constant of the torque coil was 1009.6 e.m.u., that of the quadrature coil about 246.8 e.m.u. Both were measured with precision. Each (double) coil contained 2 x 300 turns with a mean diameter of about 2.4 cm., so that its magnetic moment per ampere was about 270 e.m.u. The mutual inductance *M* between the torque coil

and the quadrature coil (as between the corresponding coils of the compensating set) was about 11 x 10^5 e.m.u. As in the earlier work the two sets of coils were connected in the circuit in such a way that their mutual inductances almost completely annulled one another.

§ 12. Induction Solenoids and Fixed Magnetizing Coils. With the hope of improving the insulation between the two circuits the upper part of the 1¼ inch induction solenoid and its compensator were for a time replaced by exactly similar coils wound on ebonite instead of bakelite. As no apparent improvement resulted (perhaps in part on account of an accident to the tube on which the induction solenoid was wound), and as the bakelite was much superior in permanence of form to the ebonite, the old coils were soon replaced.

For all work done with the magnet-mirror holders Nos. 5–7 and for some other measurements, it was necessary to remove the extension at the lower end of the induction solenoid. The removal lowered the constant of the coil (for a standard short rotor) by only a small fraction of one per cent, viz., from 216.8 to 216.3 e.m.u.

Still later, in order to make the whole suspension system shorter, and at the same time to place the rotor more nearly at the center of the fixed magnetizing coil, this coil and the induction solenoid, together with the tubes on which they were wound, were shortened, the length of the windings on each being reduced to about 37.2 cm., while the two lower ends were kept at the same level as formerly. Suitable changes were made in the compensating coil arrangement. According to experiments made with the cold-rolled steel and electrolytic iron rotors, the constant of the induction solenoid for the short rotors was not appreciably altered by these changes.

To remove any possibility of doubt of the insulation between the two layers of the fixed magnetizing coil (including the leads attached to each end), after a breakdown at the terminals had ruined a number of observations, the terminals were at first thoroughly repaired (Sept. 1934). Then, for a still greater degree of certainty, the coil was rewound with cotton tape, and later with silk tape, between the two layers. In the last case one of the layers was wound in a left hand spiral, the other in a right hand spiral. The two were connected at the bottom to pins between which a thin wire was soldered.* When the wire was removed the insulation resistance could be readily measured. Corresponding changes were made in the mutual induction compensating solenoids. The insulation in both cases always remained perfect.

In order to reduce heating, the following steps were taken. Eight wires were removed from the upper end of the induction solenoid (not enough to alter its constant appreciably), and a ring of holes bored in their place. At the same time V-shaped grooves were filed all around the edge of the lower insulating ring of amber. This made possible a steady convection current of air between the magnetizing coil and the induction solenoid. Also, holes bored laterally into the lower portion of a new amber bushing which topped the magnetizing coil and held the torsion-head made possible ventilation of the space immediately surrounding the rotor and suspension. V grooves, similar to those in the amber ring at the bottom of the magnetizing coil, were cut in the two rings at the ends of its compensator.

In order to use stronger magnetizing fields and at the same time to avoid greater forcives on the permanent magnet, still another magnetizing coil was constructed. It was 36.2 cm. long and was closely wound in four layers (without insulating tape in between) on a bakelite tube with internal and external diameters 8 and 11 mm. Otherwise the coil was similar to the magnetizing coil just described. It was mounted centrally inside a new induction solenoid, identical so far as the winding is concerned with that carrying the earlier coil, and with proper provision for ventilation. A duplicate was wound to form part of the mutual induction compensator. The new coil when producing twice the magnetic intensity of the older had a moment considerably less, and its lower end was slightly farther from the permanent magnet.

§ 13. Frequency Control. In order that the natural frequency of the rotor system might remain as nearly constant as possible throughout a complete set of vibrations, we long ago adopted the practice of establishing in the magnetizing coil long before the beginning of the set an alternating current with approximately the same heating power as that of the experimental current, but with a quite different frequency. Improve-

* Initially the connection between the two layers was made by long closely twisted wires connecting to a switching device mounted at a distance on the frame of the vertical intensity coil. In spite of the twisting, however, these wires produced a decided torque on the permanent magnet, and were removed.

ments, made some time ago, consisted in substituting a thermoammeter for another type of instrument used in equalizing the currents, and in preheating for a longer period—ordinarily four hours at least. The tuning fork also is kept in operation throughout this period.

The adjustment of the fork has been made easier by means of the additions illustrated in Fig. 5. A brass lug *B* is secured to the end of each prong *A* by means of two steel screws, one of which, *C*, is shown in the figure. The lug is threaded to hold the heavy steel screw *D* provided with a large brass head *E*. The lug is slotted, and the screw *D*, when not being turned

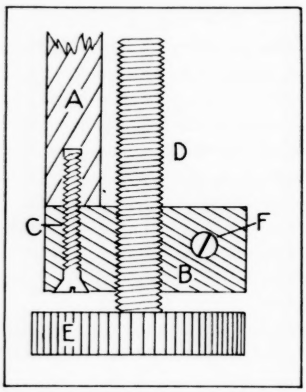

FIGURE 5. Tuning fork extension for frequency adjustment.

for frequency adjustment, is clamped by the steel screw *F*.

§ 14. THE MOMENTS OF THE ROTORS. The procedure used in determining the moments of the rotors was not discussed in III and V. In the practice of Methods II and III the magnetic moment of the rotor is needed only for estimating errors, calculating small corrections, and estimating vibration amplitudes to be expected, and need not be precisely determined. For Method II, however, exact measurements are necessary; and such measurements were almost always made even when this method was not to be used. The ballistic method of reversals was used. The rotor was hung in its standard position within the induction solenoid, or within both this solenoid and the fixed magnetizing coil, and the change of flux $\delta\varphi$ through the induction solenoid (constant $= \gamma_0$) produced by the reversal of the proper current in the magnetizing coil, either fixed or movable, was compared with the change of flux $\delta\Phi$ produced by the operation of a Hibbert magnetic standard. This latter was itself calibrated by means of a standardized ammeter and a mutual inductance standard made of the author's double solenoid* and an accurately constructed and placed secondary coil at the center. From the throws, the value of $\delta\Phi$, and the relation $\delta\varphi = 2\gamma_0 \times$ *the magnetic moment of the rotor,* this moment is determined. This moment is μ, that of the magnetic material alone, when the fixed magnetizing coil is used; or $\mu + \mu'$, where μ' is the moment of the coil wound on the rotor, when the magnetizing current is sent through this coil. The moment μ' is always calculated as the product of the total area of the winding $\times$ the current. When the fixed coil is used, care is, of course, taken to see that the mutual inductance compensation is made with the degree of precision necessary.

While the measurements of rotor moments were almost always exact, they were not in general made for the *maximum* value of the commutated current, except as needed for Method II. Often they were made for a number of different currents. The values given in the text (except when Method II is involved) are in general the moments for the *mean* value of the commutated current.

§ 15. TESTING FOR ANNULMENT OF THE EARTH'S VERTICAL INTENSITY. Inquiry has been made as to the method of testing for the annulment of the vertical component of the earth's magnetic intensity in the region occupied by the rotor.

In the earlier work, as explained in III, and in much of the later work, this was done with the horizontal intensity magnetometer and two earth inductors. One of these was mounted, at the times of primary calibration only, in the place ordinarily occupied by the induction solenoid and its supports at the center of the vertical intensity coil. The other is permanently mounted near the magnetometer and coil and serves only to compare the horizontal and vertical intensities both at the times of calibration and at the times of measuring the effect under investigation.

As it is extremely inconvenient to remove and reset the induction solenoid for repeating primary calibrations, two other methods have more recently been used. In the more satisfactory, arrange-

* A double solenoid for the production of uniform magnetic fields. Phil. Mag. **40:** 519, 1920. See also Glazebrook, Dictionary of Applied Physics **2:** 454.

ments are provided by which a comparatively small flip coil can be quickly mounted and properly operated in a position close to the center of the vertical intensity coil without disturbing the induction solenoid. Instead of using the magnetometer and comparing the horizontal and vertical intensities, it has been more convenient to cut the flux due to the vertical intensity only with the external inductor, and to use a Hibbert magnet and coil as an intermediate standard.

If Z_0 denotes the current necessary to annul the vertical intensity at the time of calibration, T_0, and Z that at the time of observation T; D_0 and D, the corresponding earth inductor throws; and S_0 and S, the corresponding Hibbert-Standard throws; we have for Z the relation

$$Z = \left(Z_0 \frac{S_0}{D_0}\right) \frac{D}{S}$$

It has never been necessary, as explained in III, to make measurements of Z_0 except at long intervals, as the proper value of Z on any occasion can be obtained from the standard determination Z_0 and the subsidiary observations made on the occasion. Throughout most of the work these subsidiary observations have been made for every group, or nearly every group, of vibration observations. But the response to small variations in Z is usually so slight as to make exact settings unnecessary, and in some of the work the observation for Z have been made only at considerable intervals, with the certainty that the slight variation of the current from its proper value for exact compensation would make no appreciable difference in the results.

§ 16. The Constant of the large Helmholtz Coils. This constant enters twice into the formula for ρ, once in determining m_0, the moment of the vibrator magnet, and once in determining Γ_0, the constant of the torque coil. An exact knowledge of this quantity is thus of great importance. As additional evidence of the correctness of its value as obtained from the absolute measurements it may be stated that this value has twice been checked by comparing the constant electrically, once in 1926 and once in 1934, with the constant of the 10 cm. subsidiary Helmholtz coil referred to in III § 34 and illustrated in Fig. 34-1 (*CC–DD*) of that paper. The constant of this coil, as calculated from its dimensions, is 39.32 e.m.u. with an error estimated as about 0.1 per cent. The earlier and later electrical comparisons with the large coil gave 39.35 and 39.33 e.m.u. respectively, thus indicating no appreciable error in the constant of the large coil.

§ 17. Corrections Associated with m_0. A slight correction to the values of m_0, and thus to the values of ρ given in III and IV, viz. the fraction $+ 0.002$, has been found necessary because the earlier magnetometer observations for lack of proportionality between deflection and intensity were made without the control magnet in position, while it was present during the regular observations for determining m_0. Also, a small correction, viz. the fraction $- 0.003$, has had to be made to a few of the results because in determining m_0 for them a certain bakelite support for the magnet-mirror holder was used. The width of this holder was determined shortly after it was made, and the result was used in the distance calculations. Later it was found that the support had shrunk slightly, a fact making the calculated values of m_0 and ρ a little too high.

There is, of course, always an error in m_0 due to the fact that when it is measured the axis of the magnet system cannot be set *exactly* horizontal. Also there is an error due to the fact that during the vibration experiments, the mean position of the axis is not *exactly* perpendicular to the axis of the torque coils. The first error makes the calculated value of m_0 (and therefore that of ρ) too small; the second makes the calculated value of ρ too large. The errors due to the inexactness of these settings were both, in general, so small as to be quite negligible; moreover, as just indicated, they tend to counteract each other.

A further fractional correction $+ 0.001$ has been applied because of a new determination of the distance between the edges of the grooves in the Helmholtz coil holder used in measuring the distance r between magnet centers while m_0 is being measured.

§ 18. Repeated Determinations of Constants. As indicated in III, V, and VI, and in part of what precedes in this paper, all of the constants involved in the determination of ρ from equation (4-2) have been determined repeatedly; and they have been determined so precisely that it is believed no residual errors in them can be responsible for errors of any consequence in ρ.

In this connection it should be said that in the recent work the separate determinations of the magnetic moment m_0 have been even more elaborate (ca. 25 per cent) than for III, and that in the course of the whole investigation over 180 determinations of this quantity have been made—in the mean about one for every three complete

sets of observations for ρ recorded in the tables, of which there are more than 550.

Other constants which are important but which do not enter into the calculations are those of the coils whose currents X, Y, and Z compensate the N–S, E–W, and vertical components of the intensity of the earth's magnetic field. Alterations, or suspected alterations, with the time of the frames on which they are wound; and alterations or suspected alterations in the magnetic field of the laboratory due to time, to shifting of iron in connection with work going on in the building, and to the erection of a new building immediately adjacent, have made necessary repeated determinations of these constants. In the course of the whole investigation the X and Y coils have been calibrated 26 times, the Z coils 6 or 7 times.

C. Further Consideration of Sources of Error in This Work

§ 19. The Possibility of Error due to Extraneous Torques upon the Magnet-Mirror Holder due to the Torque and Quadrature Coils. The magnet-mirror holder, which hangs in the alternating field of the torque and quadrature coils, is subject to an Elihu Thomson torque on accounty of asymmetry. This has no component of the fundamental frequency of the suspended system if the system is at rest. When the system is oscillating, however, it may be shown that there is a component with the fundamental frequency. This is doubtless exceedingly small, and if appreciable in this work would give values of ρ dependent upon the amplitude; whereas no such difference has been observed. A direct electrical effect seemed possible, though improbable. To test for any extraneous effect of the coils the following experiments were made.

The magnets were removed from holder No. 4 and the Cobalt-Iron rotor mounted and tuned by altering the compensation of the earth's field while the rotor magnetizing coil was traversed by the usual magnetizing current. This was then replaced by a current of the same magnitude at 50 ~/sec, which produced no oscillation but maintained the temperature unchanged. The torque coil was then traversed by currents of 50 and 100 milliamperes at the fundamental frequency. Only three or four mm. double amplitude resulted whether the system was mounted symmetrically or turned through an angle of about 2°. The current normally traversing the torque coil in the principal experiments is of the order of 10^{-5} ampere—that is, of the order of 10^{-4} the currents used in the above experiment. Thus in the actual research no possible appreciable effect could be produced in this way. Similar experiments were made with a thin Hipernik rotor replacing the Cobalt-Iron rotor, and the effects were very much smaller with a current of 100 milliamperes, whether torque coil or quadrature coil (which had a much greater constant) was used. The observed effect is probably due largely if not entirely to the residual horizontal permanent moment of the rotor.

§ 20. Torques Exerted by the Fixed Magnetizing Coil on the Magnet-Mirror Holder and Its Permanent Magnet. When the later of the 2-layer fixed magnetizing coils is traversed by a current of 0.35 ampere (producing within the coil a magnetic intensity near to 20 gausses, designated by $H = S$ in this paper) it produces at the position of the permanent magnet on holder No. 3, No. 4 or No. 8 the intensity ca. 0.0095 gauss. At the position of the magnet in Nos. 5, 6, 7 it produces the intensity ca. 0.07 gauss. The intensities due to the other fixed coils traversed by the same current are still less.

These intensities are vertical for perfect symmetry, but otherwise have horizontal components, whose in-phase effects are eliminated, when appreciable, by the reversals already described.

The vertical intensity produces on the fixed magnet a torque about a horizontal axis, which, on account of asymmetry, may produce an axial vibration. The effect of such a torque is not eliminated by either of the reversals made in this work. If appreciable, it should be much greater for the short magnet-mirror holders than for the long; and any in-phase effect should show in a difference between results obtained with the two.

To investigate the magnitude of the torques involved for imperfect symmetry the following experiments were made. The brass rotor (without winding) was suspended with each of the holders Nos. 4 and 5 in succession and driven at resonance by the torque or quadrature coils while the standard current 350 milliamperes traversed the 2-layer magnetizing coil. The torque and the amplitude were measured when the holder was displaced from the position of symmetry as much as practicable (about 3 mm.) in the direction normal to the axis of the magnets. Then the current was annulled in the torque or quadrature coil, while it continued to traverse the magnetizing coil, and the amplitude was again measured. From the two experiments the torque due to

asymmetry is readily determined. In the case of No. 4 no appreciable amplitude resulted in the second experiment, showing that the torque was less than 10^{-5} dyne cm. In the case of No. 5 the corresponding torque was about 4 or 5×10^{-4} dyne cm. When the regular gyromagnetic measurements are in progress, the magnet is never displaced from the axis of the magnetizing coil by more than a small fraction of a mm., so that the torques involved are very much less than those just given.

If the vertical intensities given above were horizontal, and normal to the axes of the magnets, they would produce the approximate axial torques 66×10^{-4} and 500×10^{-4} dyne cm., the exact values of course depending on m_0.

§ 21. Extraneous Torques on Rotor's Magnetism due to Quadrature and Torque Coils and to Fixed Magnetizing Coil. A. Quadrature and Torque Coils. In III § 12 it is shown that there is no appreciable torque on the rotor due to the action of the torque coil on its permanent magnetization, even when the center of the torque coil was only 20 cm. below the lower end of the rotor, a distance which in most of the work (with magnet-mirror holders Nos. 3 and 4) has been greatly exceeded. The torque due to the quadrature coil is usually much less than that due to the torque coil.

These torques are negligible even with the torque and quadrature coils E and the magnet-mirror holders Nos. 5–7, in the use of which the distance from the lower end of the rotor to the axis of the coils is only about 16 cm.

Thus the intensity at the lower end of the rotor due to the torque coil bears to the intensity it produces at the center of the coil, where the vibrating magnet with moment 0.7 e.m.u. is placed, the approximate ratio $2700/16^3$ to 1013, which is less than 7×10^{-4}. This coil, with magnet-mirror holder No. 5, was used with the 2.4 mm. nickel rotor, whose moment alternated between plus and minus 327 e.m.u. in the principal experiments. If we make the *extreme* assumption that the rotor is permanently magnetized to this value, and that on account of asymmetry the axis of magnetization makes an angle as great as 1/200 radian (1 mm. in 20 cm. of length) with the vertical, and if we further assume that the asymmetry is in a plane normal to the axis of the coil, the ratio of the direct torque produced on the rotor by the coil to the torque it produces on the permanent magnet with moment 0.7 e.m.u. (equal to the gyromagnetic torque when the vibration is annulled) will be

$$\frac{7 \times 10^{-4} \times 327}{0.7 \times 1013 \times 200} = 1.6 \times 10^{-6}$$

The effect of the coil on any cross-magnetization due to the full horizontal intensity of the earth's field is still less.

B. Fixed Magnetizing Coil. (1) In addition to other effects produced by this coil, attention is called in VI to a torque which seems to be quite appreciable in some of the recent work described here, viz., a torque on the rotor about a horizontal axis due to the action of the alternating vertical intensity on the permanent part of the horizontal moment, ξ. This produces a relative displacement of the rotor and suspension and thus, on account of the absence of complete symmetry about the vertical axis, gives rise to a torque about this axis which may have any phase relation with the gyromagnetic torque g. It has the frequency of the current and does not change sign or magnitude with the reversal of the system in azimuth. Its mean tends to disappear when the rotor is repeatedly mounted at random, but its existence may be responsible for some of the observed differences for which no other explanation has been found.

(2) There is also another torque, not mentioned in VI, which may possibly affect appreciably the results of all others who have worked with the fixed coil arrangement, but which is eliminated in the author's work by the asymmetry reversal I–II. It is a couple about a horizontal axis due to the action of the vertical intensity of the alternating field on the horizontal alternating moment ν. By reason of asymmetry and on account of the suspension displacement there results a couple about the vertical which has the frequency of the current and may have any phase relation with g. It tends to vanish with many settings of the rotor at random, but can be eliminated from results obtained with one setting only by the reversal I–II.

§ 22. Negligible Effect of Mutual Induction between Quadrature and Torque Coils. As is stated in V, one effect of any mutual induction between the quadrature and torque coil systems is to produce in the induction, or secondary, circuit an electromotive force (and current), and therefore a torque, nearly in phase with (or in opposition to) the gyromagnetic torque; and such an effect would have been negligible in the work described there even if the mutual induct-

ances of the two parts of the systems had been made to add up instead of to compensate one another. Here it will be shown, in a somewhat different way, that the effect (even with the mutual inductances acting together in the circuit) is negligible even in the most unfavorable cases in the new work.

Thus, if the quadrature coils were traversed by a current with amplitude I, and the two sets of coils were connected so that their mutual inductances would add instead of subtract, the ratio of the e.m.f. induced by the quadrature coils in the induction circuit to that induced by the changing moment of the rotor would be $2M \omega I/\gamma_0\omega\mu = 2MI/\gamma_0\mu$, where M, as in V, designates the mutual inductance of one pair of the coils.

The most unfavorable case in the new work is that of the pure cobalt rotor when used with the torque-and-quadrature coils C. In this case $2M = 34 \times 10^6$, I (maximum value) $= 3 \times 10^{-7}$, $\gamma_0 = 216$, and $\mu = 73$, all in e.m.u. This gives for the ratio the number 0.00065.

Much of the new work has been done with the torque-and-quadrature coils E, for which $2M$ has the much smaller value 22×10^5 e.m.u. A very unfavorable case is that of the 2.4 nickel rotor ($\mu = 327$) for which the maximum value of the quadrature current was 9×10^{-7} e.m.u. Thus the maximum value of the ratio for this rotor is about 3×10^{-5}. When these coils were connected for mutual inductance compensation, as in the measurements of the gyromagnetic ratio, the ratio of the torques was not greater than a minute fraction of this quantity, itself entirely negligible.

§ 29. Torque on the Rotor Possibly due to the Presence of Electric Charges. On one occasion, while working with the long 2 mm. wound permalloy rotor, we observed an amplitude of the order of 1 cm. which may have been due to a torque of this kind. It appeared only when the moving coil was in use, and then only when but one end of the coil was connected to the circuit. When the two ends were interchanged the phase of the torque reversed. It was always in quadrature with the gyromagnetic torque, and it disappeared when the circuit was closed even through a small condenser permitting only a current of one or two milliamperes to flow. The torque finally disappeared and we did not succeed in observing it again. For some time after this, and in order to prevent the possible accumulation of charges in the neighborhood of the rotor, we adopted the practice of inserting in a suitable receptacle projecting downward from the torsion head a small quantity of a salt of uranium. Later the practice was discontinued. To fix the relative potentials we have often made the connection between the primary and secondary circuits indicated by the dotted line in Fig. 1. But we have never observed any effect on the vibrations of making or breaking this connection.

§ 30. Possibility of Error due to the Inductive Action of the Vibrating Magnet on the Secondary Circuit. It is desirable to show that this error, which was not mentioned in III, is negligible in this work.

Let Γ_0 denote, as before, the constant of the torque coil, m_0 the moment of the magnet, and θ the angle between the axis of the magnet and the planes of the turns of the coil. Then, since θ is always very small, the flux φ sent through the torque coil by the magnet is

$$\varphi = \Gamma_0\, m_0\theta$$

The flux Φ sent through the induction solenoid (constant γ_0) in the same circuit by the rotor, with moment μ, is

$$\Phi = \mu\gamma_0$$

The ratio R of the electromotive force induced by the change of φ to that induced by the change of Φ is

$$R = \frac{\Gamma_0 m_0\, \dot{\theta}}{\gamma_0 \dot{\mu}} = \frac{\Gamma_0\, m_0\, \bar{\theta}}{\gamma_0 \bar{\mu}}$$

$\bar{\theta}$ and $\bar{\mu}$ being the amplitudes of the angular motion and the moment of the rotor. The first electromotive force is clearly opposite to the second, and in phase with the gyromagnetic torque. It makes the apparent value of ρ too large by the fraction R, which, in this work, is always far less than the experimental error, as may be seen by making the proper substitutions in the formula.

§ 31. Estimate of Error due to the Brass Tube. That no appreciable error is introduced by the presence of the brass tube in which a rotor is sometimes encased may be shown as follows.

Let D denote the mean diameter of the tube, h its thickness, l its length, all in cm., and let σ denote its resistivity in ohm-cm. Then its resistance in ohms for a cylindrical current sheet about its axis is

$$R = \sigma\pi D/lh$$

Let $\mu = \mu_0 \sin \omega t$

denote the moment of the rotor in e.m.u., and φ

the magnetic flux through the tube in e.m.u. Then, approximately,

$$\varphi = 4\pi\mu/l = \frac{4\pi\mu_0}{l} \sin \omega t$$

The electromotive force in volts induced in the tube is

$$E = \dot{\varphi}/10^8 = \frac{4\pi \cdot \mu_0\omega \cos \omega t}{l10^8}$$

Thus the current in amperes is approximately

$$I = E/R = \frac{4h\omega\mu_0 \cos \omega t}{\sigma D 10^8}$$

and the magnetic moment of the current sheet, in e.m.u., is

$$m = I\frac{\pi D^2}{4 \times 10} = \frac{\pi D h\omega\mu_0}{\sigma \times 10^9} \cos \omega t$$

$$= \frac{2\pi^2 \nu D h \mu_0}{\sigma \times 10^9} \cos \omega t = m_0 \cos \omega t$$

where m_0 is the amplitude of the moment of the current sheet and ν is the frequency in cycles per second.

The ratio of m_0 to μ_0 is

$$\frac{m_0}{\mu_0} = \frac{2\pi^2 \nu D h}{\sigma \times 10^9}$$

If we assume a brass tube with $D = 0.34$, $h = 0.056$, and $\sigma =$ ca. 0.4×10^{-6}, together with a frequency $\nu = 10$, we obtain the result

$$m_0/\mu_0 = 0.0094$$

The effect of m, which is in quadrature with μ, is thus to produce in the induction circuit only a small electromotive force in comparison with that produced by μ. And this is in quadrature with the gyromagnetic torque, and thus ineffective.

§ 32. In-phase Torques due to the Normal Action of the Quadrature Coil on the Permanent Magnet. In V, § 6, three possible in-phase effects of the quadrature coil are considered, and it is shown that two of them are always negligible. The only one which can ever be appreciable is that arising from the fact that the wave forms are not strictly flat-topped and rectangular. Neither the time-constant of the magnetizing circuit nor that of the torque coil circuit is zero, and the two differ from one another; so that there is bound to be a shift of phase from pure quadrature. Let the amplitude of the gyromagnetic torque be denoted by G, that of the full torque due to the quadrature coil by Q, and that of the in-phase (or in opposition) torque due to the quadrature coil by I. Q/G is usually, but not always, small; and its mean (algebraic) value $\overline{Q}/G$ for all the parts of a set of observations is often much smaller. Moreover, I is in general a small part of Q, so that I/Q is often negligible. When Q/G is very large, however, and sometimes when it is not large, and especially at higher frequencies, the time constants and their difference may be so large in relation to the half-period of the vibration that I/G is quite appreciable, and the apparent values of ρ depend on $\overline{Q}/G$ in an unmistakable way. When $\overline{Q}$ is large it is sometimes variable from set to set, and a graph can be drawn between the apparent value of ρ and $\overline{Q}$, from which the true value of ρ can be obtained, as the intercept on the axis $\overline{Q} = O$. Illustrations of this procedure will be found below.

In the greater part of this work the quadrature coil has not been used at all. Such a coil was first used by de Haas, in his work at Leyden, and with very low frequencies, for which it is much more suitable than for others.

§ 33. The Possibility of Error due to a Difference between the Effects of Gyromagnetic Drive and Torque Coil Drive. This subject has already been alluded to in § 10, where reasons are given for making alterations in the type of construction of the vibrating system. In the methods used in this work the rotor is driven in part by the gyromagnetic torque, which acts throughout the volume of the rotor, and in part by the torque produced by the torque coil on the group of permanent magnets, which have an entirely different location in the system. In the theory it is assumed that the two torques, for equal magnitudes, have equal effects. If they do not, different values of ρ should be obtained for holders with such different constructions as those of Nos. 2–8; and with any one holder different resonance frequencies should be obtained with separate gyromagnetic drive and torque coil drive.

Repeated experiments have shown, however, that the resonance frequency is identical whether the tuning is done by means of the gyromagnetic torque, by the torque coil, or by altering the degree of compensation of the earth's magnetic field while the magnetizing coil is in action. Our applied frequencies are remarkably constant, and the rotors are usually tuned to about one part in

15,000* of their natural frequencies. Also, no certain differences have been found in the values of ρ for the different constructions, or for the different methods of tuning. For many of the observations the tuning has been done with the gyromagnetic torque; and for many, with the torque coil drive or with the imperfect compensation referred to above. If there were any difference, gyromagnetic tuning or imperfect compensation tuning should favor the amplitude A_∞, torque coil tuning the amplitude A_2. The calculated value of ρ should then be too great for the first and too small for the second, as an inspection of the formula will show.

§ 34. Stretching Weights and Parasitic Frequencies. In addition to its natural frequency for axial vibrations, each vibrating system has natural frequencies about horizontal axes, these being more numerous the greater the number of joints in the system. To prevent interference between these frequencies and the axial frequency it is necessary to choose a suitable stretching weight; as indicated in III, since the spurious frequencies are functions of this weight. Altogether we have used six weights, ranging in mass from 183 grams to 1,032 grams, some of them being hollow to make their moments of inertia large. For a long time we have regularly tested the suitability of a weight for given rotor, mirror holder, and suspensions by measuring the time for the amplitude of free axial vibration to decay to one half. This half-time may be only a few seconds for one weight, while it may be increased to 20, 30, or even 60 seconds in some cases by substituting a different weight. We have sometimes measured these spurious frequencies by impressing a horizontal magnetic intensity on the complete rotor, magnetized with a direct current, and studying the response as a function of the applied frequency. For a given rotor these frequencies are, as would be expected, approximately proportional to the square root of the weight. They are of course highly damped, and they are lower than they were at first thought to be. In some other cases we have simply looked for the small peaks in a long range resonance curve taken by means of the magnetizing current. Usually we have simply depended on making the half time large.

D. The Observations and Results

§ 35. The Tables. The chief results of this work are given in a number of tables which will be found below in their appropriate places. H designates the magnetic intensity; the intensity $H = S$, near 20 gausses, being the standard intensity. MMH designates magnet-mirror holder; T.C. torque coil; A_∞ the (rough) mean amplitudes (in cm.) produced by the gyromagnetic torque and the effects in phase with and in opposition with it; A_0 the minimum (quadrature) amplitude; ν the frequency of the suspended system in cycles per second; μ the magnetic moment of the magnetic material of the rotor in e.m.u; μ' the moment of the coil (if present) wound on the rotor, also in e.m.u.

ρ designates the gyromagnetic ratio, or ratio of the angular momentum of the magnetic element to its magnetic moment; e/m, the ratio of the charge to the mass of an electron, the new value 1.757×10^7 e.m.u. being adopted. The gyromagnetic ratio of a (Lorentz) spinning electron being m/e, the column $\rho e/m$ gives the value of the gyromagnetic ratio of the element in terms of the ratio for the spinning electron on the assumption that only one kind of magnetic element is present.

E and W refer to the two azimuths of the permanent magnet of the suspended system, north pole east and north pole west; I and II to the two positions of the asymmetry reversing switch. Under the heading E–W, the figure I means the quantity obtained by subtracting the value of $\rho e/m$ for azimuth W from the value of $\rho e/m$ for azimuth E, both for the position I of the asymmetry switch. Similarly, the figure II means the quantity obtained by subtracting the value of $\rho e/m$ for azimuth W from its value for azimuth E, both for the position II of the switch. $\overline{\text{E–W}}$ is the mean of I and II, and $|\overline{\text{E–W}}|$ the mean magnitude.

Under the heading I–II, the letter E means the quantity obtained by subtracting the value of $\rho e/m$ for position II of the switch from its value for position I, both for the azimuth E. Similarly, the letter W means the quantity obtained by subtracting the value of $\rho e/m$ for position II from its value for position I, both for the azimuth W. $\overline{\text{I–II}}$ is the mean of E and W; and $|\overline{\text{I–II}}|$ the mean magnitude.

$\overline{\text{E–W}}$, $|\overline{\text{E–W}}|$, $\overline{\text{I–II}}$ and $|\overline{\text{I–II}}|$ have the same meanings in all tables, whether under the headings indicated above or not.

Half Sets of Observations. In the case of some of the very recent observations the calculations, instead of being made for whole sets as in Table 1, were made for half sets, as follows. The

* Or even to 1 part in 30,000.

observations of (1) A (coupled with values of A_0 taken on both sides of it) were combined with those of (2) B (together with values of A_0 taken on both sides of it) and one value of $\rho e/m$ thus obtained. A second value of $\rho e/m$ was obtained by combining in the same way the observations of (1) B (together with values of A_0 obtained on both sides of it) and those of (2) A (together with values of A_0 on both sides of it). All calculations for each half set were thus referred to the same mean time. This process was adopted both because the precision of much of the later work seemed to warrant a reduction of the number of observations in a set and also because it gave an excellent check on the assumption made in the elimination of errors that the changes occurring were proportional to the time. In the tables the half sets can be distinguished from the others by the fact that two half sets taken together bear the same date.

In addition to the great number of new observations on gyromagnetic ratios the results of which are included in the tables of this paper, a few others showing discrepancies not certainly explained completely are mentioned in the text.

§ 36. REJECTED OBSERVATIONS. The results of a number of observations have been excluded because they have little or no value on account of the unfavorable or uncertain circumstances under which the observations were obtained. Thus some have been rejected because quadrature amplitudes for at least one of the two azimuths (NPE and NPW) were too great to make reliable calculations possible; others, because of irregularities arising from fluctuation in the earth's magnetic intensity, from defects and maladjustments in the mechanical and electrical equipment, or from other sources. Some have been rejected because the in-phase (or in-opposition) torques of the quadrature coil were too large or too nearly constant (or the quadrature data otherwise inadequate) to make possible the extrapolation or interpolation necessary to eliminate their effects; others, because of short-circuits or bad contacts which developed in the electrical circuits and were not discovered until after the observations were over; or for other equally valid reasons.

The results of some of these observations, together with those of many rough and brief determinations made in the preliminary adjustments and tests of the rotors, might have been valuable in the early days of work on gyromagnetic phenomena, but they are now quite useless beside the great body of more reliable results.

§ 37. PERMALLOY. A summary of the results obtained on this material prior to August 8, 1929, when an important improvement was made in the elimination of errors, is given in Table 3 together with a summary of the results obtained on iron in the same period. All the results are reduced to the new value of e/m (1.757 e.m.u. instead of 1.77 e.m.u.) and for slight magnetometer corrections referred to above.

Table 4 contains important details of the results on permalloy between August 8, 1929 and January 12, 1930, of which only a still briefer summary was published in III; and Table 5 gives further details for the observations between May 23 and August 12, 1930, which were made, like all the later observations, on a much more rigorous time schedule.

While III was in the press the results of a few sets of observations on a 3.2 mm. permalloy rotor were added, more details for which will be found in Table 7 below.

These results are all remarkably consistent and seemed to leave little room for doubt as to the true value of $\rho e/m$ for permalloy. Nevertheless, after V, on cobalt and nickel, was completed it appeared desirable to seek a more precise answer to the question discussed in the next paragraph; and permalloy, on account of its comparative freedom from magnetostriction, was considered the best material to proceed with.

§ 38. EXPERIMENTS TO DETERMINE WHERE THE GYROMAGNETIC TORQUE IS APPLIED. As often pointed out before (first by O. W. Richardson), the torque measured in all experiments on the Einstein and de Haas effect is the combined torque on the rotor and the magnetizing coil. Theory indicates that the torque on the coil is zero,* and most experimenters in this field have measured the torque on the rotor only. In my own earlier researches, for greater certainty (and because certain undoubted sources of error can be avoided only by this procedure) the coil was usually wound on the rotor (as in the work of de Haas), so that the total torque was certainly measured. In others it was fixed to the earth. But, as pointed out in III, no certain difference was found between results obtained in the two ways, either with permalloy or with iron. The results with the coil fixed to the earth, however, were all obtained before some of the precautions taken in the later part of the work had been adopted. It therefore seemed desirable to make

* For a rigorous demonstration see F. Coeterier, Helv. Phys. Acta **8**: 522, 1935.

TABLE 3

Gyromagnetic Ratios for Iron and Permalloy Obtained Prior to Aug. 8, 1929

Group	Method	Iron		Permalloy	
		No. Sets	$\rho e/m$	No. Sets	$\rho e/m$
I	Graphical-Null	18	1.049±0.010	30	1.059±0.013
II	Independent torque-coil and gyromagnetic drives			1	1.052
III	Large and small amplitudes	41	1.035±0.007	29	1.038±0.006
IV	Large amplitudes	3	1.037±0.011	12	1.043±0.008

TABLE 4

Results on 2.4 mm. Permalloy Obtained with Large Amplitudes and Approximate Elimination of Effects of Half-Cycle Asymmetry in Each Set. Moving Coil, $\nu = 10$, T.C.A. MMH No. 3. $H = S$ and $\frac{1}{4}S$

Date	$\lvert\overline{\text{E–W}}\rvert$	$\overline{\text{E–W}}$	$\lvert\overline{\text{I–II}}\rvert$	$\overline{\text{I–II}}$	H	No. Sets	$\rho e/m$
Aug. 8, 1929–Jan. 12, 1930	0.034	−0.007	0.207	+0.033	S and $\frac{1}{4}S$	26	1.048±0.011

TABLE 5

Results on 2.4 mm. Permalloy Obtained with Large Amplitudes and Complete Elimination of Effects of Half-Cycle Asymmetry in Each Set.* H = S; T.C.A.; MMH. No. 3. $\nu = 10$. Moving coil

Date 1930	E–W			I–II			$\rho e/m$	ΔZ†
	I	II	Mean	E	W	Mean		
May 23	+0.024	−0.010	+0.007	+0.138	+0.104	+0.121	1.048	0
May 28	−0.040	+0.039	0.000	+0.050	+0.129	+0.090	1.039	0
May 29	−0.037	+0.042	+0.002	+0.120	+0.199	+0.160	1.043	0
May 30	+0.045	−0.031	+0.007	+0.169	+0.093	+0.131	1.042	0
June 4	+0.024	+0.022	+0.023	+0.099	+0.097	+0.098	1.041	0
June 7	+0.026	+0.052	+0.039	+0.184	+0.210	+0.197	1.037	0
June 9	−0.031	+0.047	−0.008	+0.046	+0.024	+0.035	1.047	0
June 25	+0.027	+0.039	+0.033	+0.076	+0.084	+0.080	1.043	0
July 12	−0.024	+0.007	−0.008	−0.089	−0.058	−0.074	1.041	0
July 14	−0.024	+0.016	−0.004	+0.045	+0.085	+0.065	1.050	+6.0
July 17	+0.051	−0.031	+0.010	−0.204	−0.286	−0.245	1.047	0
July 22	+0.040	+0.001	+0.020	−0.235	−0.274	−0.254	1.050	0
July 25	+0.041	−0.032	+0.004	+0.227	+0.144	+0.186	1.043	0
Aug. 5	+0.007	+0.004	+0.002	−0.099	−0.102	−0.100	1.049	0
Aug. 9	−0.011	+0.005	−0.003	−0.191	−0.175	−0.183	1.043	0
Aug. 10	−0.056	+0.022	−0.017	−0.192	−0.114	−0.153	1.049	0
Aug. 12	+0.010	−0.044	−0.017	−0.114	−0.169	−0.142	1.042	0

$\lvert\overline{\text{E–W}}\rvert = 0.028$; $\overline{\text{E–W}} = +0.006$ $\lvert\overline{\text{I–II}}\rvert = 0.133$; $\overline{\text{I–II}} = -0.002$ Mean $\rho e/m = 1.044\pm0.003$

* One set, judged to be defective on account of a short-circuit, was rejected.

† ΔZ is the difference between the current in the coil compensating the vertical intensity of the earth's field and its value (ca. 94) for complete compensation.

a series of more precise observations in which the rotor was driven by the fixed and the moving coil in alternate sets.

Such observations, moreover, are very important for the additional reason that they furnish the only means of investigating the magnitude of the error which is discussed in § 21, B(1) and for the elimination of which from any set on a rotor not completely symmetrical no experimental procedure has been found.

For use in measurements with the fixed coil (as explained in III) the stationary magnetizing

solenoid A (Fig. 1) was kept fixed in its normal position within the induction solenoid B, and connected in series with a similar solenoid A' within a coaxial coil B' similar to the induction solenoid, but longer, and having a few extra turns in series wound over the outside. By properly connecting the coils together and adjusting the axial position of A' with respect to B', the effect of mutual induction was made to vanish when the rotor was absent and a current of 350 milliamperes (corresponding to an axial intensity $H = S$ near 20 gausses) traversed A and A'. The setting was made to a fraction of a millimeter; while a displacement of 2 cm. produced an effect equal to only about 0.1 per cent of that of the rotor. The setting for one position of the switch which was reversed when the azimuth of the suspended system was changed between NPE and NPW differed by only 5 mm. from that obtained several years earlier when the coils were new, and before the torque-coil-quadrature coil mutual inductance compensator had been introduced. After the first five sets with the fixed coil, however, it was found that the mutual inductance between the torque coil compensator (placed in the secondary circuit when the quadrature coil was introduced) and the outer fixed solenoid in the primary circuit did not quite vanish; and the total mutual inductance was thereafter adjusted to zero for each azimuth separately.* The difference between galvanometer throws for the two positions of the switch was about 1 part in 1,100 of that due to the rotor alone.

It has been found repeatedly that the setting for compensation is independent of the time during which the current has been flowing, as would be expected on account of the similarity of the two sets of coils, although the temperature distribution is not identical in the two.

Between November 8, 1933 and January 16, 1934, 21 sets of observations were made on the short 2.4 mm. permalloy rotor, with two layers of wire, the current in alternate sets traversing the coil attached to and moving with the rotor and the coil fixed to the earth. The results are given in Group I of Table 6. An entirely unexpected difference of about 1 per cent between the two groups immediately became apparent, the persistence of which made it necessary to extend the number of sets far beyond the limits planned. The difference cannot be explained by failure of the mutual inductance compensation, as it was found unaltered to less than 0.1 per cent in December, 1933, when this series was nearly completed. The only apparent source of error is that of § 21, B (1), which may be appreciable in the fixed-coil results.

These observations were not made to determine the absolute value of ρ, but to see whether an unexpected appreciable difference would be found between the values obtained with the two methods of measurement—magnetizing coil fixed to the earth and magnetizing coil rigidly wound on the rotor and moving with it. Hence it was unnecessary to know m_0 with precision; and all the calculations are based on the mean of two closely concordant values of m_0 obtained shortly before and after Group II. The last previous determination was made on March 8, 1933, eight months before Group I began. Linear interpolation between the value then observed and the value for Group II just referred to would give for Group I a mean value of m_0 one-half per cent higher, and this would raise the values of ρ for this group by that much. The probability is, however, that the proper value of m_0 for this group is nearer that adopted, viz., that for Group II, since the results show no certain variation with the time.

Not long after the completion of Group I the suspension pulled out of the rotor, which was further injured in certain tests. It was then rewound and otherwise thoroughly repaired, and remounted. Also, torque coil B was replaced by torque coil C, and great pains were taken in an attempt to improve the insulation, defects in which, it was thought, might possibly explain the discrepancy above noted. The mutual inductances were balanced, and a number of observations were obtained. They gave about the same fractional difference between the values of ρ for the two arrangements which had been obtained before. Absolute values were not needed, as explained above, and could not be calculated, as the constant of the torque coil was not determined till a good deal later—near the end of Group II. When the constant was determined, however, it appeared that the absolute values were all much higher than those of Group I. What is undoubtedly the true explanation of the difficulty was found only after the completion of Group II, in which it did not occur, and the first series on pure cobalt. It then became manifest that a partial failure of the insulation of the primary coil had occurred just at the terminals of the leads. When the leads were moved from

* Later the outer coils were so placed and oriented that their mutual inductance completely vanished.

TABLE 6

2.4 mm. Permalloy. 1933–1935. MMH. No. 3; ν = 9.6* and 10†

Group	Date	Mag. Coil	H	T.C.	No. of Sets	$2\times A_\infty$	$\overline{\lvert E-W\rvert}$	$\overline{E-W}$	$\overline{\lvert I-II\rvert}$	$\overline{I-II}$	$\rho e/m$
I	Nov. 15, 1933, to Jan. 15, 1934	Fixed	S	B	10	cm. 3.1–5.2	0.067	−0.003 ±0.002	0.071	+0.016 ±0.017	1.037 ±0.007
	Nov. 8, 1933, to Jan. 12, 1934	Moving	S	B	11		0.017	−0.007 ±0.004	0.058	+0.050 ±0.025	1.047 ±0.004
II	June 20–22, 1934	Fixed	S	C	2	3.1–4.3	0.129	+0.015 ±0.009	0.128	+0.028 ±0.000	1.031 ±0.003
	June 19–21, 1934	Moving	S	C	2		0.005	−0.003 ±0.002	0.024	−0.024 ±0.006	1.043 ±0.001
	June 26–28, 1934	Fixed	½S	C	2	2.3–3.5	0.264	−0.008 ±0.004	0.264	+0.075 ±0.039	1.035 ±0.007
	June 25–27, 1934	Moving	½S	C	2		0.005	−0.003 ±0.002	0.024	−0.024 ±0.006	1.040 ±0.002
	July 3–9, 1934	Fixed	¼S	C	3	1.8–2.3	0.154	−0.001 ±0.005	0.154	+0.044 ±0.021	1.027 ±0.001
	July 4–10, 1934	Moving	¼S	C	3		0.005	−0.001 ±0.001	0.009	−0.008 ±0.008	1.039 ±0.005
III	Jan. 23–24, 1935	Fixed	S	A	2	4.0–4.7	0.022	−0.004 ±0.022	0.056	+0.057 ±0.005	1.059 ±0.001‡
	Jan. 15–25, 1935	Moving	S	A	4	4.0–5.2	0.020	−0.005 ±0.007	0.035	+0.028 ±0.013	1.052 ±0.018‡
	Feb. 6–18, 1935	Fixed	S	A	3	3.6–4.6	0.015	−0.012 ±0.008	0.054	−0.023 ±0.046	1.034 ±0.004
	Feb. 7–15, 1935	Moving	S	A	3	3.9–4.5	0.020	−0.011 ±0.010	0.038	−0.008 ±0.030	1.028 ±0.005

Mean $\rho e/m$ (fixed coil) = 1.037 ± 0.005 Mean $\rho e/m$ (moving coil) = 1.044 ± 0.004 Mean $\rho e/m$ = 1.040

* For the 1933–34 observations. † For the 1935 observations.

‡ These sets were made with the earth's vertical intensity doubled; the others, with all components annulled, as usual.

In Group I the mean values of the minimum, or quadrature, double amplitude were 0.40 cm. and 0.37 cm. for the fixed coil and moving coil arrangements respectively.

their usual position in a certain way, as they doubtless were during the changes above referred to, a partial short-circuit occurred. The mutual inductance balance, made not long before these observations began, was not repeated, or the source of the trouble would probably have been found much earlier.

With the new torque and quadrature coil systems in position, and with the new induction solenoid and its compensator, both wound on ebonite, replacing the corresponding coils wound on bakelite, the mutual inductance was again adjusted to zero, and remained unchanged (to at least 1 part in several thousand) when again tested on July 16. Within this interval, from June 19 to July 10, further observations of the same kind as before were made. The results of the 14 sets are given in Group II of the same table, and are essentially identical with those of the first group although currents equal to 1, ½, and 1/4 times that previously employed were used.

For the persistent small difference between the values of ρ resulting from the two experimental methods in Groups I and II, the only explanation found is the possible presence of the error discussed in § 21, B (1) in the fixed coil measurements. The attempt has of course been made to explain it by an insulation defect, as above, but the mutual inductance tests and the absolute values are both decidedly against such an explanation.

Still another series of observations on the same rotor was begun on January 15, 1935, after the primary coils had been rewound with additional precautions for insulation. The mutual inductance was first compensated with precision as before; and it remained compensated, with negligible error, throughout the series. The results are given in Group III of the table already referred to. Here there is a difference of 0.6–0.7 per cent between the two sets of results, ρ being *greater* for the fixed coil, instead of *less*, as in the earlier work. This is quite consistent with the other results as the sign and magnitude of the error mentioned above depend on the symmetry of the rotor and the way it is mounted.

The mean value of $\rho e/m$ for all of these 47 sets is close to the mean value obtained for permalloy earlier. The value obtained with the moving coil is identical with that of Table 5.

Many additional observations made later on permalloy, iron, nickel, cobalt-iron, and cobalt, as will appear below, have likewise shown only small differences between the results with the two arrangements. In general they are entirely unsystematic, and appear to be no greater than those between different groups with the same arrangement. On this account results obtained with the two arrangements have usually been given the same weight, though the error of § 21, B (1) is not present when the moving coil is used. It may account for the large discrepancies between certain of the results for pure cobalt (See p. 434).

§ 39. ADDITIONAL OBSERVATIONS WITH THE 2.4 MM. PERMALLOY. In 1937 three brief series of observations were made with the same rod under different conditions. The results are given in Table 7. For series I the rotor was unmodified, but mounted with different lower suspension and mirror-holder, No. 4.

For series II it was mounted with the short holder No. 5. Although in this series the rotor behaved excellently, and showed very slight asymmetry of any sort, faint striations appeared in the first 6 sets. It seemed impossible that these could be due to such causes as usually produce

TABLE 7

2.4 MM. PERMALLOY. FIXED AND MOVING COILS. 1937. $H = S$

SERIES I. MMH. No. 4; T.C.E.; $\nu = 18.5$. $2A_\infty = 2.1$–3.6 CM.; QUADRATURE COIL OUT

Date	Mag. Coil	E–W			I–II			$\rho e/m$
		I	II	Mean	E	W	Mean	
May 26	Fixed	−0.058	+0.060	+0.001	−0.180	−0.062	−0.121	1.024
May 27	Moving	+0.041	−0.058	−0.008	+0.455	+0.356	+0.406	1.021

Mean $\rho e/m = 1.022 \pm 0.002$

SERIES II. MMH. No. 5; T.C.E.; $\nu = 10.5$; QUADRATURE COIL OUT

Date	E–W			I–II			Fixed Coil	Moving Coil	Double Amplitude
	I	II	Mean	E	W	Mean			
July 10	+0.018	+0.028	+0.023	−0.003	+0.007	+0.002	1.059		
July 12	+0.003	+0.006	+0.004	+0.002	+0.005	+0.004		1.056	
July 13	+0.010	+0.002	+0.006	+0.008	0.000	+0.004		1.058	4.4–6.0 cm.
July 14	−0.001	+0.005	+0.002	−0.005	+0.001	−0.002	1.052		
July 15	+0.008	+0.014	+0.011	−0.008	−0.002	−0.005	1.052		
July 16	+0.001	+0.002	+0.002	0.000	+0.001	0.000		1.049	
							Mean 1.054	Mean 1.054	
July 20*(1)	−0.010	+0.010	0.000	−0.004	+0.015	+0.006	1.055+		6.7–7.0 cm.
July 20*(2)	−0.018	+0.003	−0.008	−0.016	+0.005	−0.006	1.052+		

$|\overline{E\text{–}W}| = 0.009$; $\overline{E\text{–}W} = +0.005$. $|\overline{I\text{–}II}| = 0.005$; $\overline{I\text{–}II} = 0.000$. Mean $\rho e/m = 1.054 \pm 0.003$

* After removal of spiral leads.

SERIES III. MMH. No. 6; T.C.E.; $\nu = 11.5$; $2A_\infty = 0.9$–1.7 cm.

QUADRATURE COIL IN, BUT $\bar{Q}/G = -0.002$ ONLY. FIXED COIL

Date Oct. 19	E–W			I–II			$\rho e/m$
	I	II	Mean	E	W	Mean	
(1)	+0.139	−0.165	−0.013	+0.417	+0.013	+0.215	1.058
(2)	+0.162	−0.262	−0.050	+0.376	−0.048	+0.164	1.051

$|\overline{E\text{–}W}| = 0.182$; $\overline{E\text{–}W} = -0.032$. $|\overline{I\text{–}II}| = 0.213$; $\overline{I\text{–}II} = +0.190$. Mean $\rho e/m = 1.054 \pm 0.004$

striations, and they were soon traced to the leads, which had been formed into a spiral of unusually large diameter the last time the rotor was rewound. When the leads were cut off and removed the striations completely disappeared. As the results show, the value of ρ was unaffected.

For series III the ends of the rotor were removed and replaced with the threaded flanges for the rigid attachment of the mirror holder. Holder No. 6, whose flanges did not have parallel surfaces, was used. This doubtless accounts for the large asymmetry and the small amplitude.

For reasons not apparent, series I gives a lower value of ρ, the other two series a higher value, than usually obtained with this rotor. The mean differs but slightly from that previously obtained.

§ 40. THE 3.2 MM. PERMALLOY ROTOR. Three groups of observations were made on this rotor. The results of the first group, which seems much the best, are given in Table 8. As used for this group, the rotor was of the standard long type, and was wound with four layers of wire, which the magnetizing current traversed. The mean value of $\rho e/m$ for this group was inserted in III while in the press.

For the second group of observations, Table 9 and Figure 6, the rotor was cut off, restraightened, and rewound with 2 layers of wire, to make a standard short wound rotor like the 2.4 mm. permalloy. The group can hardly be considered as reliable as the others because there were too many variables present. In the figure the apparent value of $\rho e/m$ is plotted against the quadrature coil torque necessary to reduce the quadrature amplitudes to approximately zero. There is marked dependence of $\rho e/m$ on the torque. There is no apparent dependence on the vertical compensation.

For the third group of observations, the rotor was unwound, straightened, and provided with

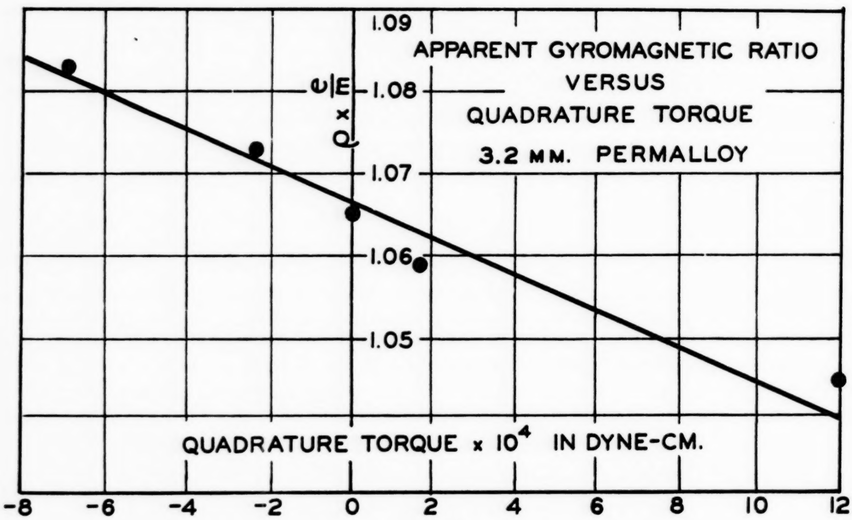

FIGURE 6.

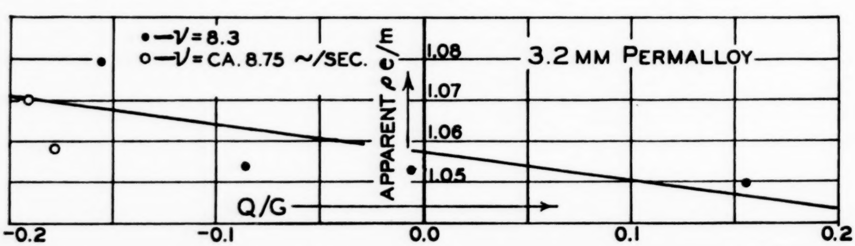

FIGURE 7. Apparent gyromagnetic ratio for 3.2 mm. Permalloy *versus* quadrature torque.

the upper and lower terminals with screws and flanges, and was used with holders No. 6 (Series I) and 7 (Series II). The results are given in Table 10 and Fig. 7. Although the departures from the mean line of the graph are greater than in the case of the second group, the result 1.058 for $\rho e/m$ is probably much more trustworthy than the larger value obtained in that group.

§ 41. The Long Wound 2.0 mm. Permalloy Rotor.* This rotor was of the standard long type, and was wound with 4 layers of No. 40 wire. Five series of observations (44 sets in all) were made, and the results are summarized in Table 11. Half the observations were made with an impressed frequency 3 times that of the rotor, and 5 sets were made with the earth's vertical field doubled in intensity by reversing the compensating current. The mean value of $\rho e/m$ for the 44 sets, is 1.052. No certain differences exist between the results obtained under the different conditions.

The detailed measurements of the amplitudes

TABLE 8

Long Wound 3.2 mm. Permalloy. $H = 2S$; $\nu = 19.3$; $\mu = 1719$; $\mu' = 6.7$; $2A_\infty = 4$–7 cm.

T.C.A. MMH. No. 3

Date 1931	E–W			I–II			$\rho e/m$
	I	II	Mean	E	W	Mean	
March 5	+0.030	−0.009	+0.010	+0.038	−0.001	+0.018	1.046
March 10	+0.110	+0.030	+0.070	+0.085	+0.005	+0.045	1.050
March 11	+0.057	+0.081	+0.069	+0.010	+0.034	+0.022	1.038
March 12	+0.052	+0.043	+0.048	−0.030	−0.039	−0.034	1.034
March 13	−0.010	−0.006	−0.008	+0.012	+0.016	+0.014	1.037
March 14	+0.033	−0.020	+0.006	+0.012	−0.041	−0.014	1.040
March 18	+0.008	+0.018	+0.013	+0.008	+0.018	+0.013	1.033*
March 21	−0.037	−0.110	−0.074	+0.205	+0.132	+0.168	1.047†
Apr. 3	+0.085	−0.068	+0.008	+0.044	−0.109	−0.032	1.064*
Apr. 4	——	——	——	——	——	——	1.030†

$|\overline{E\text{–}W}| = 0.090$. $\overline{E\text{–}W} = +0.016$. $|\overline{I\text{–}II}| = 0.047$. $\overline{I\text{–}II} = +0.022$. Mean $\rho e/m = 1.042 \pm 0.007$

* Earth's vertical intensity under-compensated by ca. 1 part in 94.

† Earth's vertical intensity over-compensated by ca. 1 part in 94.

TABLE 9

Short Wound 3.2 mm. Permalloy

$H = S$; $\nu = 18.8$; μ (moving coil) $= 1058$; $\mu' = 2.8$; μ (fixed coil) $= 1095$. $2A_\infty = 4.3$–7.7 cm.

T.C.E. MMH. No. 5

Date 1937	Mag. Coil	E–W			I–II			*Current in Z Compensator	$\overline{Q}/G$	Apparent $\rho e/m$
		I	II	Mean	E	W	Mean			
July 1	Fixed	+0.080	+0.088	+0.084	+0.005	+0.013	+0.009	92.7 div.	−0.034	1.073
July 2	Moving	+0.066	+0.087	+0.076	−0.008	+0.013	+0.002	85.2	0.000	1.065
July 3	Fixed	−0.255	−0.214	−0.234	−0.004	+0.037	+0.016	85.2	+0.170	1.044
July 6	Moving	+0.038	−0.035	+0.002	+0.014	−0.059	−0.022	94.5	−0.098	1.082
July 7	Fixed	+0.017	+0.108	+0.062	−0.042	+0.049	+0.004	102.7	+0.025	1.059

$|\overline{E\text{–}W}| = 0.099$; $\overline{E\text{–}W} = -0.002$. $|\overline{I\text{–}II}| = 0.024$; $\overline{I\text{–}II} = +0.022$. Mean $\rho e/m$ (See Fig. 6) $= 1.066 \pm 0.003$

* The vertical or Z component of the earth's magnetic intensity was compensated for a current of about 94 divisions.

* This somehow came into the laboratory as *nickel*, and in V the early observations on it were attributed to that substance. Through some curious coincidences the error was not at once detected; but the magnetic behavior of the material finally convinced me that an error had been made; and this was verified by unwinding the rotor, examining its surface color, and measuring its density.

for one complete set of observations with this rotor are given in Table 1, as an example of Method I.

§ 42. Collected Results for Permalloy and Weighted Mean Value of $\rho e/m$. In Table 12 are collected the mean results for $\rho e/m$ as given in the preceding tables and figures, with the exclusion of Table 3. For each rotor relative weights have been assigned to the different groups (except that in the case of the 2.0 mm.

TABLE 10

3.2 mm. Permalloy, Short, without Winding. $H = S$. T.C.E. $\mu = 1095$

Series I. MMH. No. 6. $2A_\infty = 1.5$–3.7 cm. $\nu = 8.3$

Date 1937	E-W			I-II			$\overline{Q}/G$	Apparent $\rho e/m$
	I	II	Mean	E	W	Mean		
Oct. 25	+0.084	−0.368	−0.142	+0.400	−0.052	+0.174	+0.156	1.050
Oct. 26	+0.144	−0.384	−0.120	+0.217	−0.310	−0.046	−0.156	1.079
Oct. 27	+0.044	−0.210	−0.083	+0.322	+0.068	+0.195	+0.017	1.060
Oct. 28	+0.058	−0.168	−0.055	+0.165	−0.061	+0.052	−0.006	1.053*
Oct. 30	+0.023	−0.261	−0.119	+0.299	+0.015	+0.157	−0.086	1.054*

$|\overline{E-W}| = 0.174$; $\overline{E-W} = -0.104$. $|\overline{I-II}| = 0.191$; $\overline{I-II} = +0.106$

* Graphical method used.

Series II. MMH. No. 7. $2A_\infty = 4.0$–5.9 cm.

Date 1937	E-W		I-II		ν	$\overline{Q}/G$	Apparent $\rho e/m$
	I	II	E	W			
Dec. 11	+0.316	−0.255	+0.410	−0.161	8.7	−0.19	1.068
	+0.340	−0.265	+0.405	−0.200			1.071
Dec. 16	+0.527	−0.579	+0.446	−0.660	8.8	−0.18	1.062
	+0.696	−0.617	+0.688	−0.625			1.055
Means	+0.470	−0.429	+0.487	−0.412			1.064±0.005

Mean E-W = +0.020. Mean I-II = +0.038

For the mean value of $\rho e/m$ obtained from I and II, viz. 1.058±0.006, see the chart of Fig. 7.

TABLE 11

2.0 mm. Long Wound Permalloy. Moving Coil. MMH. No. 3. $H = 2S$. $\mu = 638$. $\mu' = 4.4$

Series	Date	T.C.	No. Sets	$2A_\infty$	ν	$\lvert\overline{E-W}\rvert$	$\overline{E-W}$	$\lvert\overline{I-II}\rvert$	$\overline{I-II}$	$\rho e/m$	Notes
I	Dec. 24, 1930–Feb. 21, 1931	A	11	2.5 cm.	10	0.010	−0.004 ±0.004	0.022	−0.009 ±0.020	1.040 ±0.005	Impressed frequency $= 3\nu$
II	Dec. 8, 1931–Jan. 9, 1932	A	11	2	10	0.030	−0.015 ±0.011	0.076	−0.025 ±0.065	1.066 ±0.003	Impressed frequency $= 3\nu$
III	Apr. 17, 1932–May 6, 1932	A	15	6	10.3	0.011	−0.005 ±0.005	0.130	+0.130 ±0.032	1.051 ±0.003	Observations made with the Earth's vertical intensity doubled.
IV	Jan. 6, 1935–Jan. 14, 1935	C	5	4.2–4.8	9.8	0.017	+0.003 ±0.008	0.029	+0.023 ±0.021	1.057 ±0.000	
V	Feb. 24, 26, 1935	A	2	4.0–4.5	10.4	0.014	−0.003 ±0.013	0.049	+0.049 ±0.032	1.046 ±0.010	

Mean $\rho e/m$ (44 sets) = 1.052

rotor a straight mean has been taken), and a mean value for each rotor thus arrived at. If now relative weights 3, 2, 1 are assigned to the 2.4 mm., 2.0 mm., and 3.2 mm., rotors, respectively, the final mean reached* is 1.047 ± 0.004. This result is almost identical with that derived from all the early work summarized in Table 3.

§ 43. IRON. ARMCO IRON. All the results published in III were for Armco iron. Those obtained prior to November 1929 are summarized in Table 3 above. All of these were made without the systematic reversals for half-cycle asymmetry.

The mean value of $\rho e/m$ for this complete series differs but little from that of the last.

New measurements have been made on rotors of Armco iron, cold-rolled steel, electrolytic iron, and some iron kindly sent me by Professor Honda.

The results of the new measurements on Armco Iron are given in Table 15. The values of $\rho e/m$ agree closely with those already published. The 2.4 mm. rod used in the earlier work was shortened and mounted in a brass tube and provided with flanges and screw terminals for use with the magnet-mirror holder No. 7 and the short suspensions, as described in §§ 9 and 10. In Group I the quadrature coil was used, but the results, as the table shows, are independent of the quadrature. In Group II the quadrature coil was not used, except as indicated. This is one of the groups in which an effort was made to see whether the apparent value of ρ depended on the method of tuning—by gyromagnetic drive (*G*), by torque coil drive (*T.C.*), or drive by uncompensated horizontal intensity of the earth's field. No certain difference is apparent.

TABLE 12

COLLECTED RESULTS FOR PERMALLOY AND WEIGHTED MEAN VALUE OF $\rho e/m$

Rotor	Table	No. Sets	Relative weight of group	e/m	Average error in group
2.4 mm.	4	26	1	1.048	±0.011
	5	17	2	1.044	3
	6	47	2	1.043	5±
	7 I	2	1	1.022	2
	7 II, III	10	2	1.054	3
	Mean for 2.4 mm.		(weight 3)	1.043	
2.0 mm.	11	44	(weight 2)	1.052	4±
3.2 mm.	8	10	3	1.042	7
	9	5	1	1.066	3
	10	7	2	1.058	6
	Mean for 3.2 mm.		(weight 1)	1.051	
	Weighted mean $\rho e/m$ for all =			1.047	±0.004

Table 13 summarizes the results of the first series of observations on iron in which these systematic reversals were made; and Table 14 gives further details for the final series on iron published much more briefly in III. In this series the strict time schedule finally established was observed.

For the last five sets of this series the vertical intensity of the earth's magnetic field was not completely compensated, the residual intensity being made to vary from 9.8 parts in 94 in the direction of the earth's intensity to 10.0 parts in 94 in the opposite direction. The results are plotted in Fig. 8, and show distinctly a linear relation between the apparent value of ρ and the departure from compensation.

If we give to the 14 sets of Table 13 the relative weight 1, to the 26 of Table 14 the weight 2, and to each of the groups of Table 15 with 8 and 10 half-sets respectively, the weight 1, we obtain as a final mean for Armco iron $\rho e/m = 1.032$.

§ 44. COLD-ROLLED STEEL. Only one rotor of this material was investigated. It was cut from one of the rods used in the investigation described in II. It was 3.2 mm. in diameter, of the shorter

* If the three rotors are given equal weights, the result is 1.049 ± 0.004.

TABLE 13

Results on 2.4 mm. Arco Iron Obtained with large Amplitudes and Approximate Elimination of Effects of Half-Cycle Asymmetry in each Set. $H = S$. $\nu = 10$. T.C.A. MMH. No. 3. Moving Coil

Date	$\lvert\overline{E-W}\rvert$	$\overline{E-W}$	$\lvert\overline{I-II}\rvert$	$\overline{I-II}$	No. sets	$\rho e/m$
Nov. 1929–Jan. 1930	0.069	+0.022	0.254	+0.084	14	1.035±0.008

TABLE 14

Results on 2.4 mm. Armco Iron Obtained with large Amplitudes and Complete Elimination of Effects of Half-Cycle Asymmetry in each Set.† H = ca. $S/2$–S. T.C.A. MMH. No. 3. $\nu = 10$ Moving Coil

Date 1930	E–W			I–II			H		$Z-Z_0$
	I	II	Mean	E	W	Mean	$\rho e/m$	Gausses	(Z_0 = ca 94.2)
Jan. 24	+0.016	+0.012	+0.014	−0.036	−0.040	−0.038	1.029	17	0
Jan. 25	+0.032	−0.016	+0.008	+0.008	−0.038	−0.015	1.029	17	0
Jan. 27	+0.010	+0.019	+0.014	−0.029	−0.020	−0.024	1.031	17	0
Jan. 28	−0.007	+0.004	−0.002	−0.015	−0.004	−0.010	1.031	17	0
Jan. 29	+0.008	+0.021	+0.014	+0.022	+0.005	+0.014	1.025	17	0
Jan. 30	+0.019	−0.010	+0.004	+0.040	+0.011	+0.026	1.028	17	0
Feb. 1	−0.058	+0.067	+0.004	−0.128	−0.003	−0.066	1.026	17	0
Feb. 3	+0.020	−0.010	+0.005	−0.049	−0.067	−0.058	1.024	17	0
Feb. 6	+0.051	−0.010	+0.020	−0.022	−0.084	−0.053	1.021	17	0
Feb. 9	+0.005	−0.003	+0.001	+0.013	+0.005	+0.009	1.027	17	0
Feb. 10	+0.006	+0.008	+0.007	−0.014	−0.012	−0.013	1.026	17	0
Feb. 11	−0.026	−0.015	−0.020	−0.038	−0.053	−0.046	1.029	17	0
Feb. 18	+0.023	−0.024	0.000	+0.076	+0.029	+0.052	1.029	25	0
Feb. 21	+0.005	+0.020	+0.012	+0.031	+0.046	+0.038	1.039	25	0
Feb. 22	+0.009	−0.003	+0.003	+0.015	+0.003	+0.009	1.034	25	0
Feb. 27	+0.022	−0.020	+0.001	+0.038	−0.004	+0.017	1.026	25	0
Mar. 9	−0.004	−0.015	−0.010	−0.049	−0.060	−0.054	1.032	12	0
Mar. 12	+0.021	−0.047	−0.013	+0.181	+0.113	+0.147	1.030	12	0
Mar. 15	−0.059	−0.012	−0.036	+0.165	+0.129	+0.147	1.030	12	0
Mar. 17	+0.023	+0.010	+0.016	+0.206	+0.193	+0.200	1.042	12	0
Mar. 18	−0.036	+0.007	−0.014	+0.115	+0.158	+0.136	1.029	12	0
Mar. 21	+0.019	−0.007	+0.006	+0.171	+0.138	+0.154	*1.006	17	+10.0*
Mar. 22	+0.018	+0.009	+0.014	+0.116	+0.143	+0.130	*1.039	17	− 6.2*
Mar. 27	+0.011	−0.025	−0.007	+0.154	+0.118	+0.136	*1.054	17	− 9.8*
Apr. 3	+0.035	+0.019	+0.027	+0.054	+0.100	+0.077	*1.022	17	+ 6.0*
Apr. 5	+0.003	+0.012	+0.008	+0.090	+0.099	+0.094	*1.040	17	− 6.0*

Mean |E–W| = 0.019 Mean |I–II| = 0.068 Mean $\rho e/m$ = 1.032±0.003

Mean E–W = +0.003±0.010 Mean I–II = +0.051±0.071

* See curve, Fig. 8.

† One set, judged to be defective on account of a short-circuit, is not included.

type, and was wound with 2 layers of No. 40 insulated wire. The suspension was of No. 32 German silver wire. The values of A_0 were small, and the quadrature coil was not used. Only one group of observations, consisting of 7 sets, was made. The mean results are given in Table 16.

In at least a part of the work on this rotor the band of light on the scale was faintly striated. When these striations are due to ordinary harmonics, which affect A_∞ and A_2 equally, they do not introduce any error in the determination of ρ. But if they come from magnetostriction, they affect (increase) A_∞ more than A_2, and thus tend to give too large a value of ρ. It is possible that the slight excess of the value of ρ given in Table 16 over that obtained for Armco iron is accounted for in this way; but the striations were very faint (small amplitude), and the difference

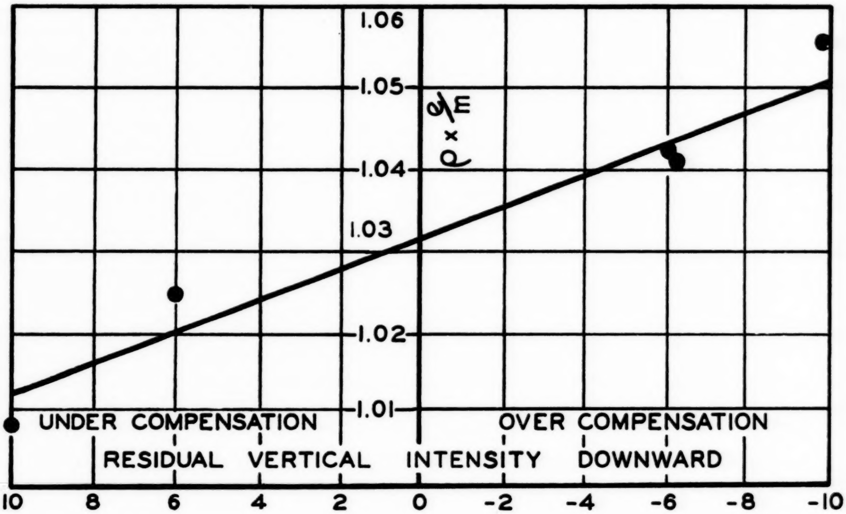

FIGURE 8. Variation of $\rho e/m$ for Armco Iron rotor with under and over compensation of the vertical intensity of the earth's magnetic field.

TABLE 15

2.4 MM. ARMCO IRON IN BRASS TUBE. FIXED COIL. $H = S$. T.C.E. MMH. No. 7. $\nu = 6.1$

GROUP I. WITH QUADRATURE COIL

Date 1937	E–W		I–II		$\overline{Q}/G$	$\rho e/m$ (4)	$\rho e/m$ (8)
	I	II	E	W			
Dec. 20*	+0.210	+0.127	+0.069	−0.014	−0.14	1.040	1.026
	+0.190	+0.115	+0.085	+0.010			1.054
Dec. 23	+0.345	+0.151	+0.070	−0.124	+0.02	1.046	1.047
	+0.430	+0.202	+0.099	−0.129			1.044
Dec. 24	+0.477	+0.066	+0.176	−0.235	−0.10	1.031	1.028
	+0.504	+0.029	+0.215	−0.260			1.034
Dec. 28	+0.289	+0.168	+0.091	−0.030	+0.03	1.006	1.005
	+0.316	+0.184	+0.061	−0.071			1.006
Means	+0.345	+0.130	+0.108	−0.106		1.031 ± 0.012	1.031 ± 0.014

* After these observations the rotor was removed to make possible a new calibration of the N–S and E–W coils neutralizing the horizontal component of the earth's magnetic intensity. It was remounted in the same way for the following observations.

TABLE 15—*Continued*

GROUP II. WITHOUT QUADRATURE COIL*

Date 1938	E–W I	E–W II	I–II E	I–II W	(8)	(4)	Tuning
Jan. 19	−0.143	+0.161	−0.436	−0.132	1.033	1.030	T. C.
	−0.157	+0.165	−0.441	−0.119	1.027		
Jan. 24	+0.045	−0.123	−0.453	−0.621	1.036	1.036	G and X
	+0.068	−0.144	−0.415	−0.627	1.035		
Jan. 26	−0.028	−0.008	−0.470	−0.450	1.038	1.038	G
	−0.025	−0.005	−0.464	−0.444	1.038		
Jan. 27	−0.168	+0.117	−0.504	−0.219	1.022	1.020	T. C.
	−0.164	+0.111	−0.486	−0.211	1.018		
Jan. 28	−0.057	−0.046	+0.198	+0.209	1.034	1.034	G
	−0.054	−0.045	+0.148	+0.157	1.033		
	$\overline{\vert E\text{–}W\vert}_{I} = 0.091$ $\overline{\vert E\text{–}W\vert}_{II} = 0.092$		$\overline{\vert I\text{–}II\vert}_{E} = 0.402$ $\overline{\vert I\text{–}II\vert}_{W} = 0.319$		Mean $\rho e/m$ 1.032 ±0.005	1.032 ±0.005	

* Except that the quadrature coil was used in finding the conductance for the minimum amplitude.

TABLE 16

3.2 MM. COLD-ROLLED STEEL. MOVING COIL

ν = 9.0. MMH. No. 3. μ = ca 1220; μ' = 2.5, e.m.u. T.C.A. MMH. No. 3

Date 1935	$2 \times A_\infty$ approx.	$2 \times A_0$ approx.	E–W I	E–W II	E–W Mean	I–II E	I–II W	I–II Mean	$\rho e/m$
July 28	6.4–6.7	0.29	−0.005	−0.007	−0.006	−0.053	−0.055	−0.054	1.059
August 12	5.4–5.6	0.39	−0.001	0.000	0.000	+0.031	+0.030	+0.030	1.043
August 14	5.6–5.9	0.64	−0.024	+0.015	−0.005	−0.054	−0.015	−0.034	1.009
August 15	5.6–5.8	0.15	−0.016	+0.007	−0.004	+0.010	+0.033	+0.022	1.046
August 18	5.7–5.9	0.24	+0.007	−0.003	+0.002	+0.039	+0.029	+0.034	1.038
August 23	5.6–5.9	0.60	−0.015	+0.018	+0.001	−0.043	−0.010	−0.026	1.038
August 24	5.4–5.8	0.24	−0.002	0.000	−0.001	−0.054	−0.053	−0.054	1.042

Mean $|E\text{–}W|$ = 0.009 Mean E–W = −0.002 Mean $|I\text{–}II|$ = 0.020 Mean I–II = −0.012

Mean $\rho/\epsilon m$ = 1.039±0.009

may be significant. In II the value of ρ for cold-rolled steel was distinctly greater than that for Armco and Norway iron.

§ 45. HONDA'S IRON. The rotor of Professor Honda's iron is referred to in V, p. 134, footnote. In its first form it was of the complex type (the magnetic material being mounted inside a brass tube, with center fixed and the ends free to slide) and was designated as Iron *C*. The iron rod was 2.0 mm. in diameter, and was turned down from a thicker rod composed of several single crystals. The brass tube was wound with four layers of No. 40 silk-enamel wire. The rotor was exceedingly difficult to work with, the in-phase and quadrature disturbing torques being very troublesome. In 1932 and 1933 three groups of observa-

tions were made on this rotor. The mean results are given in the first three lines of Table 17. The quadrature coil was used to reduce the residual amplitudes down nearly to zero. While these observations were in progress, a number of serious accidents occurred to the rotor, and it had to be repaired and remounted repeatedly. Partly as a result of this the values of A_0 (before they were reduced by the quadrature coil) were quite different in different cases and it was practicable to extrapolate from the different values of $\overline{Q}$ for the true value of $\rho e/m$. In the case of the first group the value of ρ was independent of $\overline{Q}$; but in the others, in spite of the low frequency, the dependence was quite marked.

The rotor was later modified. The rod was removed from the brass tube and out of it a standard short wound rotor was constructed. This worked so poorly that it was given up, and a new rotor of the wound complex type, but of the length of the short rotors, was constructed. The brass tube was wound with two layers of No. 40 wire. This rotor worked very well, although the amplitudes were small. The mean results are given in line 4 of Table 17. The quadrature effects were very small and the quadrature coil was not used.

The rod was later removed from the brass tube and out of it a standard short unwound rotor was constructed. Two short groups of observations were made, and the results are given in the last two lines of Table 17. Between these two groups the torque and quadrature coils were rewound and impregnated with natural ceresin wax and the insulation between primary and secondary circuits was all gone over and improved where possible by the substitution of natural ceresin wax for that used formerly. The quadrature coils were used in both series.

The mean value of $\rho e/m$ for all these observations is identical with that obtained with Armco iron. No explanation has been found for the low value resulting from the apparently excellent observations reported in the fourth line of the table.

§ 46. Electrolytic Iron. The rotor of electrolytic iron, as at first used, was similar in dimensions, winding, and mounting to that of cold-rolled steel. It was cut from the Electrolytic iron II used in the investigation described in II. Two groups of observations were made on this rotor, unaltered, in 1934 and 1936. The chief data and the results are given in Table 18. For a third group, made in 1938, the rotor was modified for the short suspensions and for rigid attachment to the magnet-mirror holder No. 7 as explained in §§ 9 and 10. The data and results for this group also are given in Table 18. The band of light in this last group was faintly striated, but so faintly that the striations were observed only with the aid of a lens. The mean value of $\rho e/m$ for the thirteen sets of observations, viz., 1.034, is almost identical with that obtained for Armco iron.

§ 47. Nickel. The earlier results on nickel rotors are summarized in Table 19, which includes the mean asymmetric effects, not published in V.

Two series of observations have since been made on another rotor, of the standard short type, unwound, 2.4 mm. in diameter. The magnetic material, like that of Nickel II, was Boker's *pure nickel* annealed. It was cut from the longer rod from which a rotor of the standard

TABLE 17

Honda 2 mm. Iron. MMH. No. 3

Date	Torque Coil	Mag. Coil	H	μ	μ'	Quad. Coil	No. Sets	ν	$2A_\infty$	$\lvert\overline{E-W}\rvert$	$\overline{E-W}$	$\lvert\overline{I-II}\rvert$	$\overline{I-II}$	$\rho e/m$
Nov.–Dec. 1932†	B	Moving	2S	907	12	on	9	5.3–5.5	2–5 cm.	0.034	+0.013	0.457	+0.457	1.041 ±0.012*
Jan.–Feb. 1933†	B	Moving	2S	907	12	on	6	5.3	1.4–5.9	0.054	—	0.414	—	1.043 ±0.015**
Mar.–Apr. 1933†	B	Moving	2S	907	12	on	6	5.3	1.1–4.4	0.102	—	0.900	—	1.038 ±0.014**
Dec. 1935–Jan. 1936‡	C	Moving	½S	474	2	off	7	6.0	1.1–1.2	0.016	+0.010	0.016	+0.010	1.002 ±0.010
Mar.–Apr. 1936¶	C	Fixed	S	590	—	on	5	9.0	6–9	0.101	+0.020	0.111	−0.206	1.034 ±0.008**
May 1936¶	D	Fixed	S	590	—	on $\overline{Q}=0$	3	9.3	5–9	0.195	−0.030	0.189	−0.044	1.035 ±0.002††

Mean $\rho e/m$ = 1.032 ±0.010

* ρ independent of $\overline{Q}$

** By extrapolation from ρ–$\overline{Q}$ curve.

† Long rotor in brass tube.

‡ Short rotor in brass tube.

¶ Shortened rotor without tube or winding.

†† Ambiguity in the quadrature coil data makes this result somewhat uncertain. On a different interpretation from that adopted the magnitude would be increased about 1 per cent.

TABLE 18

3.2 MM. ELECTROLYTIC IRON (YENSEN)

μ = ca. 1400; μ' = 2.6, e.m.u. $H = S$

Date	T.C.	MMH.	Mag. Coil	ν	Approx. $2\times A_\infty$	E–W I	E–W II	E–W Mean	I–II E	I–II W	I–II Mean	$\rho e/m$
Sept. 19, 1934	C	No. 3	Moving	8.9		−0.001	0.000	0.000	−0.018	−0.017	−0.018	1.037
Sept. 21, 1934	C	No. 3	Moving	8.9	6.7	+0.121	+0.132	+0.126	+0.030	+0.041	+0.036	1.033*
Sept. 23, 1934	C	No. 3	Moving	8.9		+0.003	+0.017	+0.010	0.000	+0.014	+0.007	1.033
Sept. 24, 1934	C	No. 3	Moving	8.9		+0.136	+0.149	+0.142	+0.005	+0.018	+0.012	1.032*
Sept. 25, 1934	C	No. 3	Fixed	8.9	to	+0.046	−0.035	+0.006	+0.035	−0.046	−0.006	1.028
Sept. 26, 1934	C	No. 3	Moving	8.9		+0.017	+0.007	+0.012	+0.009	−0.001	+0.004	1.019†
Sept. 27, 1934	C	No. 3	Fixed	8.9		+0.038	−0.050	−0.006	+0.032	−0.056	−0.012	1.025
Sept. 28, 1934	C	No. 3	Moving	8.9	8.4	−0.030	−0.018	−0.024	+0.007	+0.019	+0.013	1.014†
Sept. 29, 1934	C	No. 3	Moving	8.9		+0.025	−0.015	+0.005	+0.023	−0.017	+0.003	1.033
June 30, 1936	D	No. 4	Moving	8.12	6.0	−0.020	−0.019	−0.020	+0.087	+0.046	+0.066	1.048†
July 1, 1936	D	No. 4	Moving	8.12	6.2	−0.008	−0.005	−0.006	+0.032	+0.035	+0.034	1.040†
Jan. 17, 1938	E	No. 7	Fixed	22.6	5.5	+0.006	−0.020	−0.007	+0.046	+0.020	+0.033	1.060
Jan. 18, 1938	E	No. 7	Fixed	22.6	5.6	−0.044	+0.038	−0.003	−0.213	−0.132	−0.172	1.036

Mean $|E–W|$ = 0.027 Mean E–W + 0.020 Mean $|I–II|$ = 0.032 Mean I–II 0.000
Mean $\rho e/m$ = 1.034±0.008

* Quad. Coil used to reduce A_0. Not used in the other sets except on January 17 and January 18 while determining the resistance at which to measure minimum amplitude.

† Fixed coil in place though not used. For the remaining measurements with the moving coil the fixed coil was removed.

long type, wound, had previously been made. Repeated attempts to obtain satisfactory observations with this earlier rotor failed, probably because of the much greater difficulties encountered in making a longer and wound rotor of soft material symmetrical. The results of the two series of observations on the new rotor are summarized in Table 20, and the chief details for series II are given in Table 21. The quadrature coil was used in both series, but the values of ρ were found to be independent of the quadrature torque.

If we assign equal weights to the observations on Nickel IV and to the observations on Nickel II and Nickel III combined, and if in each group we assign double weight to the later (and better) series, we obtain, as a final mean, $\rho e/m = 1.052$. The mean result from the new work is practically identical with that obtained earlier.

§ 48. PURE COBALT. After many efforts to secure homogeneous rods of pure cobalt, suitable material was at last obtained through the kindness of Professor C. W. Drury and the Deloro Smelting and Refining Company, who brought

TABLE 19

SUMMARY OF EARLIER RESULTS FOR NICKEL ROTORS

Date—1932	Rotor	T.C.	MMH	No. Sets	ν	$2A_\infty$	$\|\overline{E–W}\|$	$\overline{E–W}$	$\|\overline{I–II}\|$	$\overline{I–II}$	H	μ	μ'	$\rho e/m$
Mar. 17–Apr. 5	Ni II	A	No. 3	6	12.8	3 cm.	0.055	+0.017	0.422	+0.394	2S	227	2.2	1.041 +0.033
Apr. 20–Sept. 19	Ni III	A. B	No. 3	17	5.4	2 cm.	0.037	+0.010	0.352	+0.352	2S	344	12	1.058 +0.018

TABLE 20

SUMMARY FOR 2.4 MM. NICKEL ROTOR (Ni IV) WITHOUT WINDING

$H = S$. $\mu = 327$.

Series	Date	MMH	T.C.	ν	$2A_\infty$	No. Sets	$\|\overline{E–W}\|$	$\overline{E–W}$	$\|\overline{I–II}\|$	$\overline{I–II}$	$\rho e/m$
I	Sept.–Oct. 1936	No. 4	D	8.01	5 cm.	11	0.128	−0.049	0.207	+0.111	1.046 ±0.018
II	Apr.–May 1937	No. 5	E	12.0	3–6	8	0.053	+0.022	0.109	+0.052	1.059 ±0.007

TABLE 21

DETAILS FOR NICKEL IV. SERIES II

Date 1937	E–W			I–II			$\rho e/m$
	I	II	Mean	E	W	Mean	
Apr. 27	+0.027	+0.012	+0.020	+0.278	+0.263	+0.270	1.049
Apr. 28	+0.033	+0.006	+0.020	+0.277	+0.250	+0.264	1.053
May 1	−0.010	−0.010	−0.010	+0.025	+0.025	+0.025	1.064
May 4	+0.076	−0.047	+0.014	+0.070	−0.053	+0.008	1.048
May 5	+0.139	−0.105	+0.017	+0.073	−0.171	−0.049	1.067
May 6	+0.110	−0.010	+0.050	−0.050	−0.170	−0.110	1.058
May 7	+0.031	+0.064	+0.048	−0.009	+0.028	+0.010	1.071
May 8	+0.014	+0.024	+0.019	−0.008	+0.001	−0.004	1.063

$|\overline{E\text{–}W}| = 0.053$; $\overline{E\text{–}W} = +0.022$. $|\overline{I\text{–}II}| = 0.109$; $\overline{I\text{–}II} = +0.052$. Mean $\rho e/m = 1.059 \pm 0.007$

my needs to the attention of the Union Minière du Haut Katanga. Through the generosity of this firm, some small plates of their pure electrolytic cobalt were provided. An examination of some of this material by his high frequency spectroscopic method has convinced my colleague, Professor G. W. Potapenko, that the impurities are less than 0.1 per cent, indicating a considerably higher degree of purity than is claimed by the company. From one of the plates were sawn a number of small rods, which were then welded together and ground by mechanicians of the Astrophysical Shop into an accurate round rod 2 mm. in diameter and of length more than sufficient to make possible the construction of a rotor of either the short or the long type. A rotor of the short type, wound with 4 layers of No. 40 wire, was constructed by Mr. Jung. Annealing did not improve the magnetic character of the material (as would be expected from the known behavior of cobalt). For a current of 100 milliamperes the magnetic moment (cobalt alone) was only about 73 e.m.u., that of the coil about 2.7 e.m.u.

With this rotor observations were started in July, 1934. The mean results of the first five sets are given in Table 22. Only the moving coil arrangement was used.

Throughout this group the quadrature coil was used to cut down the minimum amplitude. Through an accident the mutual inductance of the quadrature-coil-torque coil combination was unbalanced. As shown above and in V, however, this introduces no error. The quadrature torque, however, was very large, and the data obtained do not permit the sure elimination of any possible in-phase (or in-opposition) effect of the quadrature coil. Hence much weight cannot be attached to the results of this group

Attempts to secure even equally good results immediately after the five sets recorded for this group in the table failed on account of the breaking down of the insulation of the fixed magnetizing coil, which during these observations was kept mounted coaxially inside the induction solenoid, and supported the torsion head. A partial short-circuit occurred just where the leads were attached to the terminals of the coil, and gave rise to apparent values of ρ which were much too large.

The second group of observations, made with the fixed magnetizing coil removed, was far superior to the first and nearly perfect. The minimum amplitude all but vanished. The quadrature coil was not used. The results are given in Table 23.

A third brief but successful series, with the rotor unwound and otherwise modified for the

TABLE 22

2 MM. PURE COBALT. SERIES I. MOVING COIL

$H = 2S$: $\nu = 9.6$; MMH. No. 3; T.C.C.; $\mu = 73$; $\mu' = 2.7$; QUAD. COILS IN USE

Date	No. Sets	$2 \times A_\infty$	$\lvert\overline{E\text{–}W}\rvert$	$\overline{E\text{–}W}$	$\lvert\overline{I\text{–}II}\rvert$	$\overline{I\text{–}II}$	$\rho e/m$
July 20–26, 1934	5	0.4–0.6 cm.	0.054	−0.007	0.058	+0.020	1.079±0.016

rigid connection to the magnet-mirror-holder No. 7 and the short suspensions, was obtained in March, 1938. The results are given in Table 24. The frequency was much higher than before, and the sine-wave generator was used. The azimuthal asymmetry and the half-cycle asymmetry were both unusually great, but were eliminated by the reversals always practised in this work.

Between II and III several attempts to obtain further reliable observations failed. In some of them quadrature coil data were inadequate; also, in those made with fixed coil, the error of § 21, 13 (1) was probably very large, ξ/μ being great. The mean for all groups was similar to that for series I–III. The results for two of these groups are given in Fig. 9, where the apparent values of $\rho e/m$ are plotted against the quadrature torques $\overline{Q}$. The mean value for $\overline{Q} = 0$ is 1.08.

If we assign to the series I and III the weights 1 and 4, and to series II the weight 16, we obtain, as a final mean, $\rho e/m = 1.090$.

§ 49. Bell Telephone Copper-Cobalt. Table 25 contains the chief data and results obtained in 1932 on the complex rotor referred to in V as Cobalt IV. The table gives the asymmetry data, which were not published in V, but omits the (uncompensated) values of A_0 (made

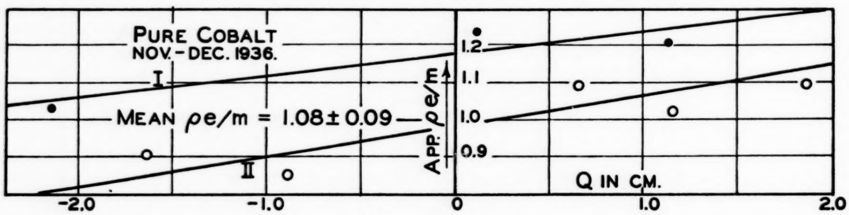

Figure 9. Apparent gyromagnetic ratio for Pure Cobalt *vs.* quadrature torque. Full circles represent observations by method I; open circles, by method II. Fixed coil in use.

TABLE 23

2 mm. Pure Cobalt. Series II. Moving Coil

$H = 2S$; $\nu = 9.6$; MMH. No. 3; T.C.C.; $\mu = 73$; $\mu' = 2.7$; Quad. Coil Out; 5 Sets

Date 1934	$2A_\infty$ cm.	$2A_0$ cm.	E–W			I–II			$\rho e/m$
			I	II	Mean	E	W	Mean	
Sept. 6	0.41–0.47	0.00–0.02	−0.005	+0.034	+0.014	−0.068	−0.029	−0.048	1.097
Sept. 7	0.61–0.64	0.01–0.03	+0.004	+0.018	+0.011	−0.033	−0.019	−0.026	1.093
Sept. 9	0.62–0.63	0.01–0.03	+0.009	+0.002	+0.006	+0.011	+0.004	+0.008	1.092
Sept. 11	0.51–0.55	0.01–0.11	−0.007	+0.035	+0.014	−0.070	−0.028	−0.049	1.096
Sept. 12	0.51–0.62	0.01–0.06	+0.002	−0.010	−0.004	−0.020	−0.032	−0.026	1.090

Means: $\overline{|E\text{–}W|} = 0.013$; $\overline{E\text{–}W} = +0.008$. $\overline{|I\text{–}II|} = 0.031$; $\overline{I\text{–}II} = -0.028$ Mean $\rho e/m = 1.094 \pm 0.002$

TABLE 24

2 mm. Pure Cobalt. Series III. Fixed Coil. Sine Wave Generator

$H = 2S$; $\mu =$ ca. 90; MMH. No. 7; T.C.E.; $\nu = 30.8$; Quad. Coil Out

Date 1938	$2A_\infty$	E–W			I–II			$\rho e/m$
		I	II	Mean	E	W	Mean	
March 9 (1)	0.130–0.463	−0.903	+0.635	−0.134	−0.905	+0.633	−0.136	1.071
March 9 (2)	0.132–0.462	−1.032	+0.652	−0.190	−0.922	+0.762	−0.080	1.108
March 12 (1)	0.157–0.430	−0.556	+0.323	−0.116	−0.702	+0.177	−0.262	1.072
March 12 (2)	0.177–0.419	−0.533	+0.422	−0.056	−0.792	−0.163	−0.314	1.054
Means		−0.756	+0.508	−0.124	−0.830	+0.434	−0.198	1.076±0.016

nearly to vanish by the quadrature coil), on which the value of ρ was shown not to depend.

In 1936 a few further observations on this rotor were made, but nothing satisfactory obtained. The rotor showed itself very unsymmetrical in I and II, and the quadrature data were insufficient to interpret. In 1937 the rod was shortened and remounted, in a (shorter) brass tube; but again the quadrature coil data and the disturbances were such as to render a reliable calculation impracticable. With three other rotors, however, successful observations were made.

Table 26 gives the data and results for a standard short rotor 2.4 mm. in diameter, without winding. The quadrature coil was used, and the quadrature torque was nearly constant in magnitude, but small. From experiments made shortly afterward on the Honda iron rotor, with variable quadrature torque and with nearly the same frequency and magnetic moment, it is probable that the quadrature effect on this cobalt rotor did not produce an error greater than ½ per cent.

Table 27 gives the data and results for a standard short rotor, without winding, 3.2 mm. in diameter. The quadrature coil was used, but the maximum value of $\overline{Q}/G$ had a magnitude of only 0.15, and the apparent values of ρ were found to be independent of $\overline{Q}$ as the table shows.

Table 28 gives the results on another 2.4 mm. rotor, mounted in a brass tube, new short style, for use with rigidly connected magnet-mirror holders Nos. 6 and 7 and the short suspensions. $\overline{Q}/G$ was large, ranging from -0.3 to -0.6, but there is no evidence of any appreciable effect on the value of ρ.

If we assign equal weights to each of the four

TABLE 25

2.4 mm. Copper-Cobalt (Long) in Brass Tube. Moving Coil.

$H = 28.$ $\mu = 1115$; $\mu' = 12.$ $\nu = 5.5.$ MMH. No. 3. T.C.B. $2A_\infty = 4.5$ cm. ±

Date 1932	E–W I	E–W II	E–W Mean	I–II E	I–II W	I–II Mean	$\rho e/m$
June 27	−0.100	−0.077	−0.088	+0.150	+0.174	+0.162	1.113*
June 28	−0.055	+0.020	−0.018	−0.004	+0.071	+0.034	1.077
June 29	−0.020	−0.032	−0.026	+0.351	+0.339	+0.345	1.063
July 5	−0.089	−0.035	−0.062	+0.216	+0.270	+0.243	1.060
July 6	−0.048	−0.041	−0.044	+0.360	+0.367	+0.364	1.076
July 11	−0.082	+0.013	−0.034	+0.030	+0.125	+0.078	1.070
July 12	−0.012	−0.074	−0.043	+0.055	−0.007	+0.024	1.059
July 14	−0.037	−0.045	−0.041	+0.163	+0.155	+0.159	1.064
July 16	−0.039	−0.063	−0.051	+0.127	+0.103	+0.115	1.077
July 19	−0.034	−0.035	−0.034	+0.233	+0.232	+0.232	1.074
July 20	+0.026	+0.064	+0.045	+0.080	+0.118	+0.099	1.070
July 27	0.000	−0.032	−0.016	+0.247	+0.215	+0.231	1.065
Aug. 4	−0.047	−0.065	−0.056	+0.120	+0.106	+0.113	1.075

Mean |E–W| = 0.045. Mean E–W = −0.036 Mean |I–II| = 0.170. Mean I–II = +0.169 Mean $\rho e/m$ = 1.073±0.009

* Adjustments of rotor and torque coil much improved after this set.

TABLE 26

2.4 mm. Copper-Cobalt (Short) Without Winding

Date 1936	E–W I	E–W II	E–W Mean	I–II E	I–II W	I–II Mean	$\rho e/m$	Other Data $2A_\infty$ = 5–8 cm.
March 9	+0.017	−0.080	−0.032	+0.261	+0.164	+0.212	1.100	MMH. No. 4
March 11	+0.032	−0.122	−0.045	+0.347	+0.193	+0.270	1.090	T.C.C.
March 13	+0.060	−0.025	+0.018	+0.259	+0.174	+0.216	1.070	ν = 7.9 ~/sec
March 14	−0.036	−0.057	−0.046	+0.278	+0.274	+0.276	1.112	μ = 486 e.m.u.

$|\overline{E\text{–}W}| = 0.054$; $\overline{E\text{–}W} = -0.026.$ $|\overline{I\text{–}II}| = 0.244$; $\overline{I\text{–}II} = +0.244.$ Mean $\rho e/m = 1.087 \pm 0.013$

TABLE 27

3.2 MM. COPPER-COBALT (SHORT) WITHOUT WINDING

$H = S$. $\mu = 679$. $\nu = 14.5$. T.C.E. MMH. No. 4

Date 1937	E–W			I–II			$\overline{Q}/G$	$\rho e/m$
	I	II	Mean	E	W	Mean		
May 17	+0.046	−0.249	−0.102	+0.844	+0.549	+0.696	−0.15	1.079
May 18	+0.251	−0.232	+0.010	+0.950	+0.467	+0.708	−0.01	1.096
May 19	−0.067	−0.008	−0.038	−0.131	−0.072	−0.102	−0.02	1.080
May 20	+0.138	−0.175	−0.018	+0.030	−0.283	−0.126	−0.01	1.096
May 21	−0.388	+0.250	−0.069	−0.005	+0.633	+0.314	−0.07	1.100
May 22	−0.390	+0.400	+0.005	−0.159	+0.631	+0.236	−0.08	1.123

$|\overline{E–W}| = 0.216$; $\overline{E–W} = -0.035$. $|\overline{I–II}| = 0.396$; $\overline{I–II} = +0.288$. Mean $\rho e/m = 1.096 \pm 0.011$

TABLE 28

2.4 MM. COPPER-COBALT IN BRASS TUBE, SHORT, WITHOUT WINDING

$H = S$. $\mu = 486 \pm$*. $\nu = 7.1$. T.C.E. MMH. No. 6

Date 1937	E–W			I–II			$\overline{Q}/G$	$\rho e/m$ (8)	$\rho e/m$ (4)
	I	II	Mean	E	W	Mean			
Nov. 22	−0.204	−0.127		−0.391	−0.304		−0.4	1.076	1.090
	−0.297	−0.107		−0.412	−0.222			1.105	
Nov. 23	−0.078	−0.212		−0.355	−0.489		−0.4	1.071	1.081
	−0.123	−0.228		−0.346	−0.451			1.091	
Nov. 24	−0.156	−0.124		−0.372	−0.340		−0.6	1.035	1.041
	−0.290	−0.047		−0.379	−0.136			1.047	
Nov. 27	−0.144	−0.226		−0.561	−0.643		−0.3	1.085	1.079
	−0.098	−0.136		−0.530	−0.568			1.073	
Means	−0.174	−0.151	−0.162	−0.418	−0.394	−0.406	−0.42	1.073±0.016	1.073±0.016

* The moment of this rotor was not measured, but the rod was of the same material and dimensions as that of Table 26, and must have had nearly the same moment.

groups of observations in Tables 25–28, we obtain as a final mean, $\rho e/m = 1.082$.

§ 50. COBALT-IRON. This rotor was at first of the standard short type. The magnetic material was 3.2 mm. in diameter, and was cut from the cobalt-iron (Preuss's alloy) compensator used in the investigation of II. It was wound with four layers of No. 40 wire, and provided with an upper suspension of No. 34 German silver wire. The first two series of observations were obtained in 1934 and 1936. Both were apparently excellent. The quadrature coil was not needed in either series. The chief data and results are contained in Table 29.

In the first series (1934) only the moving coil arrangement was used. The amplitudes were inconveniently large with the standard current of 100 milliamperes, so that all but the first set of observations were obtained with a current of one half that magnitude (corresponding to an impressed magnetic intensity of $H = S$). In the second series (1936) both moving and fixed coil arrangements were used, and six sets were made by Method II.

With the standard method, no difference appears between the results obtained by the fixed and moving coil arrangements. With Method II, which is less reliable, there is a difference of 1.8 per cent, which is far larger than the apparent error.

The inexplicable discrepancy of about 3 per cent between the results of the series I and II led to series III and IV, the chief data and results for which are given in Tables 30 and 31 and Fig. 10, I.

No certain difference is indicated between the results for fixed coil and moving coil methods. But series III and the last part of series IV, made

TABLE 29

3.2 mm. Cobalt-Iron, Series I (1934) and Series II (1936). Without Quadrature Coil

Date	T.C.	MMH	ν	H	μ	μ'	$2A_\infty$	$2A_0$	Mag. coil	$\lvert\overline{E-W}\rvert$	$\overline{E-W}$	$\lvert\overline{I-II}\rvert$	$\overline{I-II}$	No. Sets	$\rho e/m$
1934 Nov. 23–29	C	No. 3	3.16	S 2S	1158 1636	2.8 5.6	8.7–9.4 11.5–13.9	0.2–4.1 0.3–1.1	Moving	0.015	+0.001	0.034	−0.018	5	1.008 ±0.006
1936 May 19–June 16	D	No. 4	4.54	S	1274		4.2–4.9	0.0 to 0.3	Fixed	0.024	−0.011	0.024	+0.018	5	1.040 ±0.009
1936 May 23–June 18	D	No. 4	4.54	S	1158	2.8	3.6–4.8		Moving	0.026	−0.022	0.024	−0.008	6	1.038 ±0.006
1936 May 28–June 3	D	No. 4	4.54	S	1256 and 1264		4.2–4.8		Fixed	0.026	+0.024	0.012	+0.005	2*	1.048 ±0.002
1936 May 29–June 8	D	No. 4	4.54	S	1184	2.8	4.0–4.7		Moving	0.027	−0.024	0.012	−0.010	4*	1.030 ±0.006

Mean $\rho e/m$ = 1.032

* Observations made by Method II.

TABLE 30

3.2 mm. Cobalt-Iron, Series III, Without Quadrature Coil

$H = S$. MMH. No. 4. T.C.E. $\nu = 13.8$. $2A_\infty = 4.7$–8.1, $2A_0 = 0.1$–1.6

Magnetizing Coil	Date 1937	E–W			I–II			$\rho e/m$
		I	II	Mean	E	W	Mean	
Fixed	May 29	+0.002	−0.010	−0.004	−0.060	−0.072	−0.066	1.002
	June 1	−0.042	−0.008	−0.025	−0.041	−0.107	−0.074	0.995
	June 4	+0.008	+0.001	−0.004	−0.077	−0.084	−0.080	1.016*
	June 10	−0.020	+0.026	+0.003	−0.052	−0.006	−0.029	1.019
	June 12	−0.048	+0.037	−0.006	−0.086	−0.001	−0.044	1.020
Mean $\lvert$E–W$\rvert$ = 0.020		Mean E–W = +0.009			Mean $\lvert$I–II$\rvert$ = 0.059		Mean I–II = −0.059	Mean $\rho e/m$ = 1.010±0.005
Moving	May 31	+0.028	+0.010	−0.018	+0.020	+0.002	+0.011	1.015
	June 3	+0.013	−0.016	+0.029	+0.033	+0.004	+0.018	1.020
	June 9	+0.013	+0.002	+0.011	+0.077	+0.066	+0.072	1.033
	June 11	−0.001	−0.003	+0.002	+0.038	−0.003	+0.018	1.002
$\lvert\overline{E-W}\rvert$ = 0.011		$\overline{E-W}$ = +0.006			$\lvert\overline{I-II}\rvert$ = 0.030		$\overline{I-II}$ = +0.030	Mean $\rho e/m$ = 1.018±0.009

* During this set the coils of the mutual inductance compensator were both short circuited. Proper allowance was made for this in the calculations.

with the long magnet-mirror holder and at nearly the same frequency (ca. 14 cycles per second) agree almost exactly, with $\rho e/m = 1.014$ and 1.010; while the first part of series IV, made with a short holder and at a much lower frequency, gives 1.038. In series IV there is no dependence on quadrature in the case of the low frequency work, while $\overline{Q}/G$ happened to be negligible with the rotor as mounted for the higher frequency.

No explanation of the differences between the values of ρ being apparent, another series of observations was made, viz., series V, for which the results are given in Table 32, and Fig. 10, Curve II. For this series the cobalt-iron rod was mounted in a brass tube and provided with the proper terminals for rigid attachment to the magnet-mirror holder No. 7 and to the short suspensions. There is a marked dependence on the quadrature torque, the mean value of $\rho e/m$ (corresponding to $\overline{Q} = 0$) being 1.020.

If we assign to the 22 sets of series I and II and to the 23 sets of series III–V equal weights,

TABLE 31

3.2 MM. COBALT-IRON, SERIES IV, WITH QUADRATURE COIL

$H = S$. T.C.E. $2A_{\infty}$ = 6.4–10.5 (Aug. 1–7); 4.4–7.0 (Aug. 26–31)

Date 1937	Mag.-Coil	Mirror Holder	Frequency cycles	E-W I	E-W II	I-II E	I-II W	$\overline{Q}/G$	Apparent $\rho e/m$
Aug. 1 (1)	Fixed	No. 5	5.1 sec.	−0.047	+0.139	−0.159	+0.027	+0.037	1.038
Aug. 1 (2)	Fixed	No. 5	5.1	−0.062	+0.154	−0.157	+0.059		1.042
Aug. 5 (1)	Fixed	No. 5	5.1	+0.093	+0.027	+0.078	+0.012	+0.116	1.042
Aug. 5 (2)	Fixed	No. 5	5.1	+0.101	+0.020	+0.096	+0.015		1.037
Aug. 6 (1)	Moving	No. 5	5.1	+0.038	+0.043	−0.019	−0.016	−0.085	1.039
Aug. 6 (2)	Moving	No. 5	5.1	+0.036	+0.037	−0.020	−0.019		1.039
Aug. 7 (1)	Moving	No. 5	5.1	+0.047	+0.040	−0.008	−0.015	+0.021	1.035
Aug. 7 (2)	Moving	No. 5	5.1	+0.040	+0.041	−0.013	−0.012		1.035
Aug. 26 (1)	Fixed	No. 4	14.2	−0.105	+0.030	−0.201	−0.066		0.999
Aug. 26 (2)	Fixed	No. 4	14.2	−0.095	+0.038	−0.211	−0.078		1.003
Aug. 30 (1)	Fixed	No. 4	14.2	−0.085	+0.045	−0.104	+0.026	Negli-	0.993
Aug. 30 (2)	Fixed	No. 4	14.2	−0.085	+0.005	−0.094	−0.004	gible	1.014
Aug. 31 (1)	Moving	No. 4	14.2	−0.012	−0.003	−0.022	−0.013		1.021
Aug. 31 (2)	Moving	No. 4	14.2	−0.034	−0.009	−0.050	−0.025		1.030

Mean $\rho e/m$ for Aug. 1–Aug. 7 = 1.038±0.002. (See Fig. 10, I) Mean $\rho e/m$ for Aug. 26–Aug. 31 = 1.010 ± 0.012

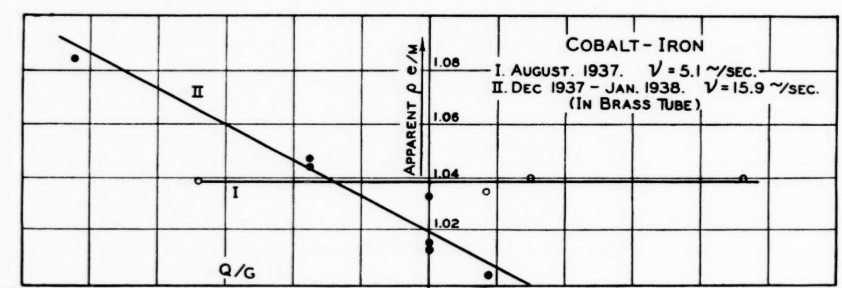

FIGURE 10. Cobalt-Iron. Apparent gyromagnetic ratio *vs.* quadrature coil torque.

and if in the latter group we assign equal weights to series III, series V, and the first division of series IV, and half as much weight to the second half of series IV we obtain, as a final mean, $\rho e/m$ = 1.027.

§ 51. HIPERNIK. Several rods of this material were kindly prepared and sent to me by Dr. Yensen and Mr. Frey of the Westinghouse Electric and Manufacturing Company, for which I am greatly indebted to them and the company. After forging, swaging, and drawing, they had to be annealed in a suspended position with weights attached to the lower ends, a very special heat treatment, in order to insure straightness. Even then the heavier rods did not come out perfectly straight. So they were straightened by a slight stretching and were re-annealed in silica tubes. The material is so soft that it has to be handled with extreme care.*

Two rotors were at first constructed from this material, one of them a standard wound rotor of the longer type 2.4 mm. in diameter, the other a standard unwound short rotor 1.6 mm. in diameter. A number of attempts to secure satisfactory observations with these rotors, especially with the 2.4 mm. rotor, were failures. With this rotor the gyromagnetic ratio was roughly obtained as early as 1928, and agreed within a per cent or two with the precise determinations obtained recently. But the observations were very difficult and rough, apparently on account

* These statements are quoted almost verbatim from Dr. Yensen.

TABLE 32

3.2 MM. COBALT-IRON, SERIES V (BRASS TUBE WITHOUT WINDING)

$H = S$. T.C.E. MMH. No. 7. $\nu = 15.9$. $A_\infty = 0.5$–3.0

Date	E–W		I–II		Q/G	Apparent $\rho e/m$	
	I	II	E	W		(14)	(7)
December 30, 1937	+1.341	+1.368	−0.036	−0.009	−0.131	1.082	1.084
	+1.253	+1.447	−0.204	−0.010		1.087	
December 31, 1937	+1.299	+1.388	−0.054	+0.035	−0.044	1.046	1.047
	+1.334	+1.360	−0.002	+0.024		1.048	
January 4, 1938	+0.020	+0.069	−0.030	+0.019	0.000	1.042	1.033
	+0.024	+0.024	+0.017	+0.017		1.024	
January 5, 1938	−0.042	+0.049	−0.096	−0.005	0.000	1.025	1.016
	+0.018	+0.074	−0.067	−0.011		1.008	
January 6, 1938	+0.103	+0.028	+0.063	−0.012	0.000	1.016	1.014
	+0.036	+0.082	−0.037	+0.009		1.012	
January 7, 1938	−0.737	−0.608	−0.147	−0.018	−0.044	1.055	1.044
	−0.644	−0.642	−0.127	−0.145		1.034	
January 8, 1938	+0.020	+0.079	−0.076	−0.027	+0.022	1.009	1.004
	−0.005	+0.080	−0.090	−0.005		0.998	

$\overline{|E\text{–}W|}_I = 0.491$; $\overline{|E\text{–}W|}_{II} = 0.521$. $\overline{|I\text{–}II|}_E = 0.075$; $\overline{|I\text{–}II|}_W = 0.025$. Mean $\rho e/m = 1.020$ (See Fig. 10, II)

of the severe magnetostriction which this material exhibits. The Page effect was exceedingly audible, and the band of light on the scale was badly broken up with the lines due to pseudo-harmonics.

When, however, the 1.6 mm. rotor was re-mounted in a brass tube in such a manner as to eliminate as far as possible the effects of magnetostriction and was used with the short upper and lower suspensions and the rigidly connected magnet-mirror holder, it behaved most beautifully, and gave the results quoted in Table 33. Magnetically, this rotor was so symmetrical that in the full earth's field the amplitude was not more than two or three times the gyromagnetic amplitude. All quadrature effects were small. The band of light on the scale was perfectly smooth. The magnetic moment of the rotor, as used with a current of 350 milliamperes, was about 370 e.m.u. The quadrature coil was used; but, as the table shows, the values of ρ obtained are independent of $\overline{Q}$.

The mean value obtained for $\rho e/m$ viz., 1.052, does not differ from that for pure nickel.

TABLE 33

1.6 MM. HIPERNIK (YENSEN) IN BRASS TUBE, FIXED COIL

$H = S$. μ = ca. 375. T.C.E. MMH. No. 6. $2A_\infty = 1.07$–1.55 CM.

Date 1937	E–W			I–II			$\rho e/m$ Half Sets	$\rho e/m$ Complete Sets	$\overline{Q}/G$	ν $\sim$/sec.
	I	II	Mean	E	W	Mean				
Oct. 11 (1)	−0.050	+0.038	−0.006	+0.003	+0.091	+0.047	1.058	1.056	+0.127	7.3
Oct. 11 (2)	−0.029	+0.033	+0.002	−0.002	+0.060	+0.029	1.054			
Oct. 12 (1)	−0.003	+0.040	+0.018	−0.139	−0.096	−0.118	1.049	1.044	−0.006	7.3
Oct. 12 (2)	−0.046	+0.089	+0.022	−0.216	−0.081	−0.148	1.040			
Oct. 13 (1)	+0.010	+0.034	+0.022	−0.124	−0.100	−0.112	1.062	1.054	−0.085	7.3
Oct. 13 (2)	+0.030	+0.005	+0.018	−0.072	−0.097	−0.084	1.045			
Oct. 14 (1)	+0.019	+0.003	+0.011	−0.077	−0.093	−0.085	1.060	1.060	+0.036	7.3
Oct. 14 (2)	+0.041	+0.003	+0.022	−0.075	−0.123	−0.099	1.060			
Oct. 15 (1)	+0.074	+0.069	+0.072	−0.038	−0.043	−0.040	1.027	1.046	−0.054	9.2
Oct. 15 (2)	+0.061	+0.101	+0.081	−0.022	+0.018	−0.002	1.065			

$\overline{|E\text{–}W|} = 0.039$; $\overline{E\text{–}W} = +0.026$. $\overline{|I\text{–}II|} = 0.079$; $\overline{I\text{–}II} = -0.070$ Mean 1.052 ±0.009 Mean 1.052 ±0.006

§ 52. HOPKINSON'S IRON-NICKEL ALLOY (IRON 75%, NICKEL 24.5%). In 1934 a rotor of this material was constructed. The rotor was of the standard short type, and wound with four layers of No. 40 wire. The magnetic material was 3.2 mm. in diameter, and was cut out of the Hopkinson alloy compensator used in the investigation of II. As explained in II, the material had been properly treated thermally to make it as highly magnetic as practicable. The upper suspension was of No. 32 German silver wire. The quadrature amplitudes were small and it was unnecessary to use the quadrature coil. Eight good sets of observations were obtained. The results are contained in Table 34 under series I. Only the moving coil arrangement was used, with an impressed magnetic field of strength $2S$.

In 1938 the rotor was reconstructed into the new standard short type without winding, and was used with the rigidly attached magnet-mirror holder No. 7 and short suspensions. The magnetic field was somewhat stronger than for series I. The results of the observations, which were so good that it was unnecessary to take many of them, are given in Table 34 under series II.

If we assign to series II the same weight assigned to series I we obtain, as a final mean, $\rho e/m = 1.024$.

§ 53. COBALT-NICKEL. In 1934 a standard short cobalt-nickel rotor was constructed. It was wound with two layers of No. 40 wire and provided with a suspension of No. 32 German silver wire. The magnetic material was 3.2 mm. in diameter, and was cut out of the cobalt-nickel compensator used in the investigation of II. Observations with this rotor were difficult, and magnetostrictive striations were always present in the band of light on the scale. The residual amplitudes A_0 was small, and the quadrature coil was used in only one set of the first series of observations, a summary for which is given in Table 35.

In 1935 work with this rotor was resumed. It was found that the torsion head was cracked,

TABLE 34

3.2 MM. ROTOR OF HOPKINSON'S ALLOY (IRON 75%, NICKEL 25%).

SERIES I, MOVING COIL

$H = 2S$. $\mu = 319$, $\mu' = 5.5$. $\nu = 5.2$. T.C.C. MMH. No. 3. $2A_\infty = 0.9$–1.5.

$2A_0 < 0.1$ EXCEPT IN FIRST SET

Date 1934	E–W			I–II			$\rho e/m$
	I	II	Mean	E	W	Mean	
Dec. 11	−0.041	+0.031	−0.005	+0.269	+0.341	+0.305	1.005
Dec. 13	+0.106	−0.130	−0.012	+0.342	+0.106	+0.224	1.019
Dec. 14	−0.001	−0.018	−0.010	+0.169	+0.152	+0.160	1.024
Dec. 17	−0.009	−0.042	−0.026	+0.321	+0.288	+0.304	1.031
Dec. 18	−0.031	−0.028	−0.030	+0.286	+0.289	+0.288	1.009
Dec. 19	−0.062	−0.021	−0.042	+0.213	+0.254	+0.234	1.007
Dec. 20	+0.025	+0.032	0.028	+0.244	+0.251	+0.248	1.026

Mean |E–W| = 0.041 Mean |I–II| = 0.252 Mean $\rho e/m = 1.017 \pm 0.009$
Mean E–W = −0.014 Mean I–II = +0.252

SERIES II. WITHOUT WINDING

$H = 2S$. $\mu = 441$. T.C.E. MMH. No. 7. $\nu = 9.9$

Date 1938	$2A_0$	$2A_\infty$	E–W			I–II			$\rho e/m$
			I	II	Mean	E	W	Mean	
April 1	0.065 to	2.2 to	−0.256	+0.266	+0.005	−0.264	+0.258	−0.003	1.030
	0.135	2.7	−0.225	+0.213	−0.006	−0.211	+0.227	+0.008	1.027
April 2	0.170 to	1.8 to	−0.504	+0.530	+0.013	−0.539	+0.495	−0.022	1.030
	0.375	3.1	−0.497	+0.530	+0.016	−0.542	+0.485	−0.028	1.036
Means			−0.370	+0.385	+0.007	−0.389	+0.366	−0.011	1.031±0.003

the insulation had become defective, and the rod had become bent. The rotor was then rewound and otherwise thoroughly repaired, and six more sets of observations obtained. Although the residual amplitudes were small, the quadrature coil was again used. The resulting values of ρ were independent of the magnitude of the quadrature coil torque. A summary of the principal data and results is given in Table 35, and further details in Table 36.

The second series is far superior to the first, but the mean results of the two differ only slightly. In this second series the amplitude of the striation harmonics was so small that the residual double amplitude $2A_0$ could be reduced by the quadrature coil to zero within two or three tenths of a millimeter.

In 1938 the rotor was reconstructed into one of the standard new short unwound rotors in a brass tube with the standard new terminals, and was used with short suspensions and the rigidly attached magnet-mirror holder No. 7. The quadrature coil was not used. Striations, as was to be expected, were entirely absent. The chief data and results are given in Table 37.

This is one of the groups in which an effort was made to see whether any difference in the magnitude of ρ would result from tuning to resonance by torque-coil drive and rotor drive, the two modes of drive being used in this group alternately in complete sets. As the table shows, no appreciable difference was found.

If we give equal weights to the last two series, and half as much to the first series, the mean value of $\rho e/m$ is 1.077.

§ 54. Heusler Alloys. Several attempts have been made to secure reliable observations on Heusler alloys, but so far with limited success. Two rotors were constructed from rods sawn from the large rotor referred to in II as *Heusler alloy I*. One was of the standard short type, unwound, the other of the short type, unwound, in a brass tube. The material was very porous and the rod of the second rotor was in two pieces, cemented together. In both cases the disturbances were too great for observations of any value.

A number of other rods were cast in vacuo and with the greatest care possible at the Research Laboratory of the General Electric Company

TABLE 35

3.2 mm. Cobalt-Nickel. Summary for Series I and II

Moving Coil. $H = S$. MMH. No. 3. T.C.C. $\nu = 9.0$

Series	Date	No. of Sets	$2A_\infty$	$\lvert\overline{E-W}\rvert$	$\overline{E-W}$	$\lvert\overline{I-II}\rvert$	$\overline{I-II}$	μ	μ'	$\rho e/m$
I	Nov. 7–15, 1934	4	1.4–3.1	0.226	−0.069±0.108	0.328	+0.290±0.288	429	2.6	1.085±0.038
II*	Sept. 27–Oct. 8, 1935	6	2.0–2.9	0.047	+0.045±0.006	0.075	+0.020±0.032	419	2.5	1.077±0.006

* Rotor rewound and restraightened.

TABLE 36

3.2 mm. Cobalt-Nickel. Series II. Detailed Results for $\rho e/m$

Date 1935	E–W			I–II			$\rho e/m$
	I	II	Mean	E	W	Mean	
Sept. 27	+0.004	+0.072	+0.038	0.000	+0.068	+0.034	1.090
Sept. 29	−0.008	+0.088	+0.040	+0.001	+0.097	+0.049	1.073
Oct. 1	+0.028	+0.071	+0.050	+0.045	+0.098	+0.072	1.079
Oct. 5*	+0.072	+0.038	+0.055	+0.005	−0.029	−0.012	1.067
Oct. 7	+0.027	+0.055	+0.041	−0.053	−0.025	−0.039	1.074
Oct. 8	+0.048	+0.057	+0.048	+0.010	+0.019	+0.014	1.078

$\lvert\overline{E-W}\rvert = 0.047$; $\overline{E-W} = +0.045\pm0.006$. $\lvert\overline{I-II}\rvert = 0.075$; $\overline{I-II} = +0.020\pm0.032$; Mean $\rho e/m = 1.077\pm0.006$

* Magnet-mirror holder straightened and reversed in azimuth with regard to the rotor.

TABLE 37

3.2 MM. COBALT-NICKEL. SERIES III. ROTOR IN BRASS TUBE, WITHOUT WINDING

$H = S$. $\mu = 451$. MMH. No. 7. T.C.E. $\nu = 6.2$. $2A_{\infty} = 0.51\text{–}1.62$

Date 1938	E-W		I-II		$\rho e/m$	
	I	II	E	W	(1) Tuning by Torque Coil	(2) Tuning by Rotor
Feb. 16	+0.879	−1.027	+0.810	−1.096	1.085	
	+0.784	−0.949	+0.707	−1.026	1.086	
Feb. 17	+0.713	−0.849	+0.655	−0.907		1.060
	+0.793	−0.817	+0.575	−1.035		1.057
Feb. 18	+0.967	−0.969	+0.711	−1.225	1.073	
	+0.850	−0.926	+0.730	−1.046	1.079	
Feb. 22	+0.897	−0.833	+0.773	−0.957		1.076
	+0.912	−0.803	+0.726	−0.989		1.061
Feb. 23	+0.780	−0.801	+0.646	−0.935	1.065	
	+0.692	−0.689	+0.575	−0.806	1.054	
Feb. 24	+0.636	−0.713	+0.480	−0.869		1.074
	+0.545	−0.642	+0.431	−0.756		1.090
Means	+0.787	−0.835	+0.652	−0.971	(1) 1.074±0.010	(2) 1.070±0.010
Means	−0.024		−0.160		(1) and (2) 1.072±0.010	

through the kindness of Mr. Fuller and Mr. Collins. To them and the Laboratory I am greatly indebted for this service. Some of these rods were of material taken from Heusler alloy I, composed of Al, Mn, and Cu in atomic proportions; some from new materials in the same proportions; some from new materials in the proportions Al, Mn, Cu_2; and some from the same metals in still different proportions with the addition of a minute quantity of lead. Attempts to give these rods suitable heat treatment to make them strongly magnetic were failures except in the case of the rods of composition $AlMnCu_2$. This material became suitably magnetic by simply quenching at 850° C., as recommended by Valentiner and Becker.* For placing the facilities of his metallurgical laboratory at our disposal for all this heat treatment, I am much indebted to my colleague, Prof. Donald Clark.

The rods of $AlMnCu_2$ were brittle, and not entirely free from holes, as X-ray examinations at Schenectady showed. In the attempt to grind the best of them into a true cylinder the rod broke into pieces. Several such pieces were then ground with flat ends in the Astrophysical Shop; and an unwound rotor, latest style, of approximately the standard short length, was constructed with great skill by Mr. Julius Pearson. Three lengths of material were cemented closely together with litharge and oil in closely fitting cylindrical brass sleeves. The rotor was hung up and tested for geometrical symmetry and was found to be exceedingly true.

In spite of this high degree of mechanical perfection, however, gyromagnetic observations showed a great deal of asymmetry in I and II. Three sets of observations were made, giving $\rho e/m = 0.96 \pm 0.12$. Further work was postponed until a change in method could be made, or until the storage battery plant of the laboratory, which was no longer in condition to drive the equipment satisfactorily, could be overhauled.

E. CONSIDERATION OF CERTAIN SOURCES OF ERROR IN THE WORK OF OTHERS

§ 55. DR. BATES' CLAIM THAT THE GYROMAGNETIC RATIO FOR FERROMAGNETIC SUBSTANCES IS EXACTLY m/e. In what precedes a large amount of conclusive evidence from measurements on the Einstein and de Haas effect has been presented which shows that the gyromagnetic ratios for many ferromagnetic substances very definitely exceed the value m/e. The extensive researches on the Barnett effect described in II had already yielded the same result, and will be discussed further below, where the conclusions from the two investigations will be compared.

In a brief article in Nature, however, Dr. L.

* Zeit. f. Phys. **83**: 371–403, 1933. I am indebted to Professor Valentiner for further advice by letter.

F. Bates* contends that the true value of the gyromagnetic ratio for ferromagnetic substances (except pyrrhotite) is exactly m/e (which is the value, within the experimental error, derived from my first work on the magnetization of iron by rotation in 1914), and therefore that the larger values obtained in my laboratories since are not correct. He bases his claim on work done by Sucksmith and himself, by Coeterier and Scherrer,† and by Ray-Chaudhuri.‡

The investigation of Ray-Chaudhuri is an extensive and accurate piece of work on a number of iron oxides. In the preliminary paper, which Dr. Bates quotes, the gyromagnetic ratio was given as only slightly greater than m/e. But the final paper has since appeared, and in it the ratio is given as $1.03 \times m/e$ and $1.02 \times m/e$ for all of the oxides, in almost exact agreement with my value for iron.

The work of Coeterier and Scherrer on iron was incidental to their work on pyrrhotite, and was apparently restricted to a single determination for each of two very small specimens, with small magnetic moments approximately equal to those of their specimens of pyrrhotite. The values obtained were $m/e \times 0.995$ and $m/e \times 1.01$. But the authors do not claim precision, they give no estimate of the error, and the data presented are insufficient to enable the reader to form an adequate independent judgment of its magnitude.

The results of Sucksmith and Bates, reduced for the new value of e/m (viz. 1.757×10^7 e.m.u.) are summarized in Table 38. Group I contains the joint work of the two investigators; group II and III, additional work by Sucksmith.

* L. F. Bates, Nature **134:** 50, 1934.

† See F. Coeterier, Helv. Phys. Acta **8:** 522, 1935.

‡ D. P. Ray-Chaudhuri, Ind. J. of Phys. **9:** 383–414, 1935.

Unquestionably these results furnish no certain evidence that $\rho e/m$ for any of the materials differs from unity. The value 1.05 for Cu-Ni alloy, however, which is quoted both by Sucksmith and by Bates as favorable to the 1.00 value for nickel, would certainly seem to support my value for nickel, which is 1.05, more than it supports the value unity.

Sucksmith and Bates believe that the total error in group I is no greater than 1 or 2 per cent. But the data published with respect to some of the constants and measurements are insufficient to enable the reader to form an adequate independent judgment of the magnitude of the possible error due to these; and there are other possible sources of error, which are discussed below.

§ 56. The Correction to the Constant of the Induction Solenoid. In making this correction Sucksmith and Bates adopted a procedure which is equivalent to assuming that a uniform solenoid wound *on the rotor, and of exactly the same length*, is equivalent to a much longer or infinite solenoid. When the diameter of the rotor is small in comparison with its length, as in their work, the assumption is approximately, but not exactly, justified. There is no merit in making the two lengths exactly equal. The exact theory is as follows:

If a coil of wire traversed by unit electric current produces throughout any region the magnetic intensity γ, then a body with magnetic moment μ inserted in this region with its axis parallel to the direction of γ sends through the coil the magnetic flux $\phi = \mu\gamma$. If the body is a short rod placed axially in a much longer solenoid, $\gamma = 4\pi n$, where n is the number of turns per unit length.

If the rod is placed in a shorter solenoid, with the same pitch, the flux is $\varphi' = \mu\gamma_0$, where γ_0

TABLE 38

Gyromagnetic Ratios from the Work of Sucksmith and Bates

Group	Material	No. Sets	$\rho e/m$	Range of $\rho e/m$	Approx. 2 × vibration amplitude in m.m.
I	Iron	16	0.998±0.012	0.966–1.020	10–20
	Nickel	6	0.994±0.004	0.980–1.002	5–10
	Heusler Alloy	14	0.994±0.006	0.980–1.006	5–8
II	Cobalt	9	1.022±0.064	0.908–1.152	1
	Magnetite	10	0.982±0.048	0.886–1.138	0.7
III	Ni-Cu Alloy (56% Ni)	6	1.05±0.10	0.95–1.18	<10

(less than $\gamma = 4\pi n$) is the *effective* constant of the coil for the particular rod in its definite position.

In the work of Sucksmith and Bates, and in most of my own, the gyromagnetic ratio ρ is determined in effect from the formula $\rho = A\gamma_0$, A including all the factors involved other than the constant of the solenoid. But in the work of Sucksmith and Bates γ_0/γ was determined by the procedure referred to above, which should give *too large* a value of γ_0, and thus of ρ.

Although the correction of any error due to this precedure would make the discrepancy between my values and those of Sucksmith and Bates larger instead of smaller, I have thought it desirable to make independent determinations of γ_0/γ for nickel and iron rods similar to those used in their work and in fields similar to theirs. I have therefore made such determinations, with a uniformly wound solenoid of length 53 cm. and of the same diameter (1.00 cm.) as that of their induction solenoid and provided with central taps distant apart the exact length of their rotors, viz. 15.2 cm.

Each of three rods of annealed nickel of this length, and with diameters 1.57 mm., 2.08 mm., and 2.30 mm., and each of three rods of Armco iron of the same length and with diameters 1.58 mm., 2.38 mm., and 3.2 mm. was in turn placed centrally and coaxially within this solenoid, and the whole mounted centrally and coaxially within a much larger magnetizing coil (A) of about the dimensions of that of Sucksmith and Bates, and traversed by a current which produced a field very nearly uniform in the region occupied by the rod and of the same strength (90 gausses) as that used by them. By several careful series of ballistic measurements the ratio γ_0/γ was determined for each of the rods.

The magnetizing coil A was made of 7 shorter cylindrical coils, all exactly alike, placed end to end on a long bakelite tube.

To vary the conditions of the tests, another set of determinations of γ_0/γ was made with the coil A lengthened by adding another of the short coils at one end, without other change. The extended coil will be referred to as coil B.

The smallest two iron rods were tested with coil A, all three with coil B. For the first two the agreement was almost exact for A and B, and γ_0/γ was very nearly a linear function of the diameter, increasing from about 0.944 to about 0.956 as the diameter increased. The interpolated value for a diameter equal to that of Sucksmith and Bates' smaller iron rod is 0.9445; for the diameter of their larger rod, 0.9452. The differences for the two coils A and B were less than the experimental error.

With coil A, γ_0/γ for the three nickel rods was found to be 0.937 ± 0.003; with the longer coil B, 0.935 ± 0.007. No certain differences were manifest between the results for the different rods.

According to these experiments, Sucksmith and Bates' γ_0/γ for nickel is too small by the fraction $0.936 - 0.923 = 0.013$. Their value for the smaller iron rods (5 sets) is in almost exact agreement with mine, the difference being 0.944–0.942, a quantity less than the experimental error. For their larger iron rods (11 sets) their value is apparently too large, the difference being $0.962 - 0.945 = 0.017$.

According to this, Sucksmith and Bates' value of ρ for nickel should be increased by 1.3 per cent; while their value for the larger iron rods should be decreased by 1.7 per cent, and thus their final mean for iron by 11/16 of this, or about 1.2 per cent. With these alterations, and the change in the value of e/m, their results for iron and nickel, respectively, would become 0.986 m/e and 1.007 m/e.

Thus such error as occurred in making the end corrections cannot explain the discrepancies between their work and that of Ray-Chaudhuri, Barnett and Barnett, and myself.*

§ 57. Other Corrections and Possible Causes of Discrepancies between Results. In the same article Dr. Bates indicates that he believes I have overestimated the importance of eliminating the effects of electron inertia, as well as the effects of a residual vertical magnetic field and of magnetostriction. Electron inertia effects are negligible in all work on this subject except that part of my own in which the magnetizing current traverses a coil wound on the rotor; but I have suspected the other effects, in association with asymmetry and half-cycle inequalities, responsible for appreciable errors in much of the work done on rotation by magnetization.

The correction for electron inertia in my experiments (appreciable only when the magnetizing coil is wound on the rotor) has ranged from 0.1 per cent for certain iron and permalloy rotors

* Since this was written, moreover, Kikoin and Goobar in a preliminary paper have published for iron at a liquid air temperature the value $\rho e/m = 1.026$. An estimate of the error is not given. See C. R. Acad. Sci. U.S.S.R. **19**: 249, 1938.

in a 2-gauss field to more than 7 per cent for a rotor of pure cobalt. While negligible, or nearly negligible, in some cases, it is by no means so in all. Moreover, the theoretical value of the correction has been completely confirmed by direct experiments on copper (See IV). The approximate fractional value of the correction is $2\ \mu'/\mu$ where μ' is the magnetic moment of the magnetizing coil and μ that of the magnetic material.

As to the vertical intensity, data are given in III and V, and additional data are given in this paper, to show that its elimination is sometimes necessary, even in the case of easily magnetizable materials like soft iron and permalloy. In many cases, where the degree of symmetry is high and the magnetizing field strong, it is doubtless immaterial whether the vertical intensity is annulled or not. But when it is not annulled evidence should be adduced to prove that it is without effect.

For evidence that the elimination of the vertical intensity is at least sometimes necessary even in the case of soft iron, see Table 14 and the graph in Fig. 8. For possible evidence in the case of permalloy see Table 6 (Group III).

Sucksmith in his work on nickel-copper alloys,† has made experiments which convince him that the vertical intensity is without effect. But the error in that work is so large (estimated by Sucksmith at 10% per cent) that the result cannot be considered generally applicable.

Dr. Bates' idea that it is only on account of possible magnetostrictive effects that I have considered it necessary to eliminate the vertical intensity of the earth's field is not correct. That part of the error which is connected with magnetostriction can be eliminated by reversing the connections to the power supply. But even when this is done, as in my own work, there is sometimes left a definite error, which can be eliminated only by destroying the vertical intensity. See III, pp. 285, 286, and VI, § 32 (c).

As to magnetostriction, I had suspected that a considerable part of the discrepancies between the work of Sucksmith and Bates and my own was due to the fact that they did not make systematic reversals of the connections between commutator (or generator) and the magnetizing coil in order to correct for asymmetry of the half-cycles of current and magnetization. In my own work great improvement immediately resulted from the adoption of this practice. Indeed, it has usually been more important to make this reversal (between switch positions I and II), than to work with two azimuths of the suspended system differing by 180°, as many of the preceding tables show.‡

§ 58. THE HALF-CYCLE ASYMMETRY REVERSAL AND THE AZIMUTHAL REVERSAL. The reversal I–II not only eliminates the effect of magnetostriction, whether the magnetizing coil is fixed to the coil or fixed to the earth, but in the latter case it also eliminates the effect of the axial torque (see III § 12 (b)) due to the action of the horizontal part of the intensity of the alternating field on the alternating horizontal moment ν of the rotor; but if the half-cycles of current and magnetization are equal, the fundamental frequency of this torque is twice that of the current, and so its effect is negligible. If the half-cycles are not equal, there is also, in general, a residual torque with the frequency of the current, as pointed out by de Haas. On the assumption that both the vertical moment μ and the horizontal moment ν of the rotor are in phase with the current, this torque is in quadrature with the gyromagnetic torque. But while the first assumption may be *nearly true* in the case of flat-topped waves, it is not *exactly* true; and the second may be far from true. This torque may thus have an in-phase component which is not negligible and may definitely require the I–II reversal.

When the magnetizing coil is wound on the rotor this torque vanishes, and the necessity of the reversal, which the tables clearly show, is probably due entirely, or almost entirely, to magnetostriction.

The reversal in azimuth, introduced by de Haas, was used by Sucksmith and Bates in order

† Nature **134:** 936, 1936 and Helv. Phys. Acta **8:** 205, 1935.

‡ Sucksmith, in the articles just referred to, states that in the work of Sucksmith and Bates no effect of magnetostriction was ever witnessed. This is doubtless due in part to the fact that they did not make the reversal I–II. Throughout my own much more extensive work, even with suspended systems apparently much more symmetrical than theirs, there is abundant evidence, of several kinds, of the effect. It is even true that rotors, both in the moving coil arrangement and in the fixed coil arrangement, can be driven, usually with large amplitudes, by applying electromotive forces with just one-half their natural frequencies. In the case of the moving coil arrangement this is entirely due to magnetostriction. As pointed out earlier, error due to this effect, like most of the others, must vanish with complete symmetry about the vertical, which never exists.

to eliminate the effect of the torque due to the action of the (fixed) magnetizing coil on the residual permanent horizontal magnetization of the rotor, and is very important. The same reversal, as de Haas states, eliminates the effect of the torque due to the action of the residual horizontal component of the earth's magnetic intensity on the horizontal component of the alternating magnetization of the rotor. This effect of the azimuthal reversal is not referred to by Sucksmith and Bates, but in the work described here at least it is of greater importance than the other. For the two effects of the reversal together are measured by the quantities E–W in my tables; and this quantity is in general of the same order of magnitude whether the rotor is driven by a coil attached to and moving with it (in which case the effect of the first torque vanishes), or whether it is driven by a coil fixed to the earth, as in all of the work of most others.

The tables in this article give many illustrations of the quantities E–W and I–II in my work, for both superior and inferior rotors. The I–II values are in general greater, and often much greater, than the E–W values, a fact which justifies the statement made above with regard to the relative importance of the two kinds of reversal.

From the data given by Sucksmith and Bates for iron, nickel, cobalt, magnetite, and Heusler alloy, I have made approximate calculations of the quantities corresponding in their work to my |E–W|. They are given in Table 39.

TABLE 39

|E–W| IN THE WORK OF SUCKSMITH AND BATES

	Minimum	Maximum	Mean
Iron	0.04	0.53	0.17
Nickel	0.07	0.45	0.28
Cobalt	0.7	6.0	1.9
Magnetite	0.65	2.4	1.6
Heusler alloy	0.27	0.74	0.45

A comparison of this table with my own tables will show that its values of E–W are in general much greater than my own. Since E–W and I–II both depend, though not in exactly the same way, on rotor asymmetry, and since my I–II values are, in general, greater than my E–W values, it would seem that the reversal corresponding to my change between I and II, would have been even more important in the work of Sucksmith and Bates.

The commutators used in my own work were all carefully made by a skillful machinist. Nevertheless the segments are not exactly equal. Thus in the single commutator with 1/8 inch gaps there is a mean difference between the lengths of the segments for the two half cycles of about 1 part in 170; in the similar commutator with ¾ inch gaps, the difference is about 1 part in 700; and in the triple commutator with 1/32 inch gaps, the difference is about 1 part in 1000.

As would be suspected, some of the trouble appears to come from the state of the brushes and the commutator surfaces or slip rings. In nearly all of the work described here a great deal of pains has been taken to keep these in good condition—in the case of the commutator surfaces, by accurate turning on the lathe, by treatment with fine sandpaper, and by cleaning with benzine or gasoline. There appears to be evidence that if the commutator surfaces are treated with fine sandpaper before *every set* of observations the asymmetrical effects are sometimes reduced. For a long time this has been our regular practice. Examination of our wave-shapes, both flat-topped and sine, with the oscillograph has shown only perfectly smooth and regular operation, at both low and high frequencies, and whether condensers and resistances were placed across the gaps or not. The oscillograph used, however, was not a sensitive one.

It is possible that in the work of Sucksmith and Bates the degree of half-cycle inequality in their electromotive forces, and the state of their contact surfaces, were such as to make the asymmetric effects less than in the work described here. The error, however, arises from these causes, coupled with the asymmetry of the rotor and the magnetizing field about the vertical; and this, to judge from the E–W values of Sucksmith and Bates, was, in general, much greater in their work than in that described here.

In the long run, with random mounting, $\overline{\text{E–W}}$ and $\overline{\text{I–II}}$ both tend to vanish; but in the work described here the results would be far less trustworthy if error were not eliminated by the practice of making *both* reversals.

§ 59. The Minimum Amplitude Test for Neutralization of the Earth's Horizontal Intensity. In the work of Sucksmith and Bates, as well as in that of some others, the currents in the coils which are provided to neutralize the earth's horizontal intensity are varied until the resonance amplitude of the rotor is a minimum, when it is assumed that the residual horizontal intensity vanishes.

This would be true if, as further assumed, and

only if, the torque* due to the earth's field were strictly in quadrature with the gyromagnetic torque, and were the only quadrature torque present. In general this is by no means true, as was in effect pointed out by de Haas, and as the E–W values for rotors driven by the coil wound rigidly on them prove. There are in general various quadrature torques present, and the minimum amplitude means only that their sum is a minimum for a certain residual horizontal intensity, which may be either small or great.

If this residual intensity remains constant, the major part of its effect is in general eliminated by observing with the suspended system in two azimuths 180° apart. But if the currents are adjusted to minimum amplitude for *each* azimuth of the system, the residual intensities are different for the two settings, and the torque due to the earth's residual field cannot be eliminated this way. It is not clear which procedure was followed in the work of Sucksmith and Bates.

It is clear from what precedes that when the field compensators are set in the way indicated the minimum amplitude observed for the apparent gyromagnetic torque may be either greater or less than that due to the gyromagnetic torque alone.

As an example from tests made here long ago, a difficult rotor known as Cobalt II (W. E. Cu Co.) gave a large residual amplitude A_0 for NPE when

* This torque is proportional to the rotor's horizontal moment ν, which may not be in phase with the vertical moment μ. Thus it may not be in quadrature with the gyromagnetic torque, which is proportional to $\dot{\mu}$. Sucksmith and Bates think of the torque as a pure quadrature torque.

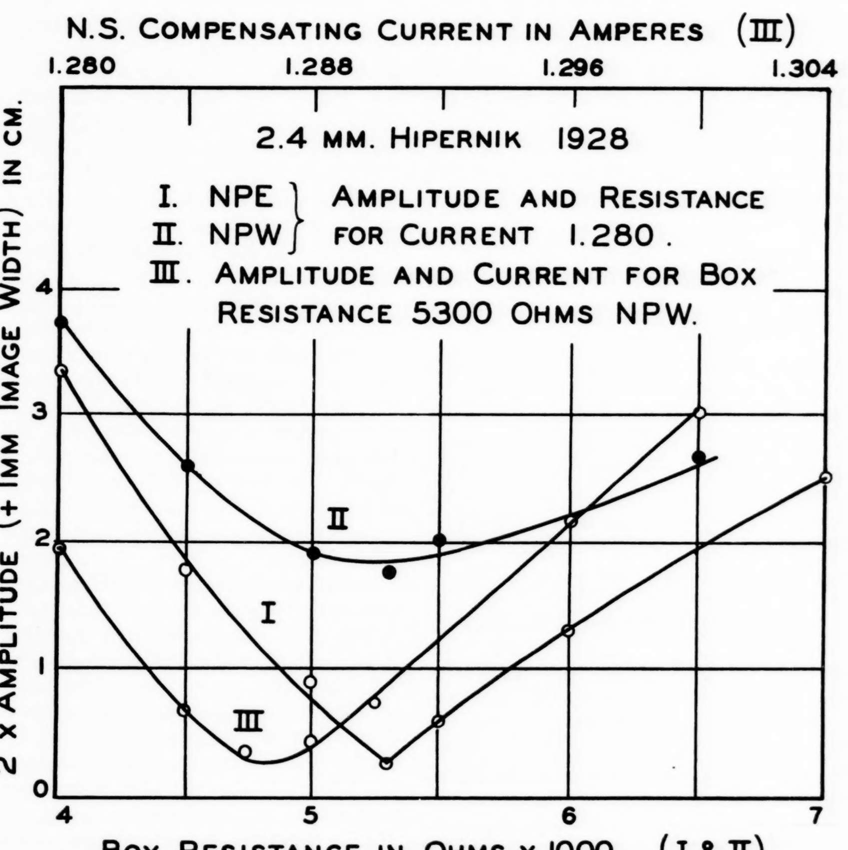

FIGURE 11.

the currents X and Y in the N–S and E–W coils were given the proper values to compensate the earth's horizontal intensity. The proper value of X was 1.263 ampere. When X was changed to 1.276 ampere, $2A_0$ was reduced to about 1 cm. (the apparent gyromagnetic amplitude being about 7 cm.). For NPW, however, this value of X gave for $2A_0$ about 10 cm. This was reduced to about 1 cm. by changing X to 1.252 ampere—about as much below the correct value as 1.276 was above it. The calculated value of $\rho e/m$, viz, about 1.05, could not be assumed correct. When compensation was properly made, larger values resulted, but A_0 was so large that little reliance could be placed upon them, and the rotor was discarded.

Fig. 11 supplies another example, from somewhat rough observations on a Hipernik rotor. With the compensating currents set at the proper values to neutralize the earth's magnetic intensity (X being 1.280) Curves I and II for NPE and NPW were obtained, the minimum amplitude for the second being far in excess of that for the first.* In order to produce as small a minimum for NPW as given with $X = 1.280$ for NPE, it was necessary to increase X by more than 0.006, as shown by Curve III.

Part III. FURTHER DISCUSSION OF THE WASHINGTON WORK ON THE BARNETT EFFECT

§ 60. Quantitative Results and Difficulties with Magnetization by Rotation. In the article by Dr. Bates already quoted he refers to my work on magnetization by rotation as having been successful since 1914, and says that the results agree with those obtained from the converse effect "as well as could be expected." But he says they will not be considered further, and he does not mention them again. Thus he gives no weight whatever to them in deriving what he considers a correct value of the gyromagnetic ratio.

In reply to an inquiry from me about this he wrote me as follows: "The reason that I did not treat your experiments on magnetization by rotation in more detail in my article in *Nature* was that I have always considered that the experimental difficulties were such that one could not reasonably expect from them more than a welcome confirmation of the order of magnitude of the ratio obtained by other methods."

Now there can be no doubt that these experiments are difficult. Before they were successful with me, *and since*, they have been tried, or planned and given up, by a number of investigators, including very eminent investigators, in England, France, and America; but they have published no results.

When Einstein and de Haas learned that this work had succeeded with me they referred to it as "incomparably more difficult than their own," and Einstein has since told me that "he never thought it could be done."

Nevertheless I believe that no one who will read the original account of even the earliest experiments, those of 1914 and 1915 by the method of electromagnetic induction, can fail to acknowledge the strictly quantitative character of the work. The results of the two (independent) early investigations agree within a few per cent with one another, and with all the best work on gyromagnetic effects which has been done since. The work which followed, by magnetometer methods, became far more extensive and precise; and if Dr. Bates had read (as I cannot believe he has read) the original accounts of it, I think it would be impossible for him to refrain from giving it great weight.

To obtain precise results elaborate precautions of many kinds were necessary. The precise rotation work was always done after midnight, the most important part between the hours of two and four in the morning. To facilitate the work the Capital Traction Company of Washington even stopped the operation of its Chevy Chase streetcars while the measurements were in progress. Nevertheless even in the daytime, with both the sun and the street cars in full operation, and in a few minutes, observations of a quantitative character could be obtained, as the next section shows.

§ 61. Observations Made in the Daytime by Visitors to the Washington Laboratory. To see this, it is only necessary to inspect Table 40, which gives all the results obtained, in each case in the daytime and in a few minutes, by visitors to the laboratory. The only observations given are the mean magnetometer deflections; but the visitors also measured the sensitivity, and one of the investigators or an assistant measured the speeds.

As would be expected, the errors are far

* The ordinates as given include the width of the image, viz., 1 mm., so that the actual minima are 1 mm. less than their curves indicate.

TABLE 40

RESULTS OF ROUGH OBSERVATIONS ON MAGNETIZATION BY ROTATION MADE IN THE DAYTIME BY VISITORS TO THE LABORATORY

Observers	Date	Rotor	No. of Scale Readings	Mean Deflection on Reversal of Rotation	Specific Magnetic Intensity of Rotation (λ)	Gyromagnetic Ratio $\rho \times e/m$
					gauss / rev. per sec.	e.m.u.
J. C. Merriam and L. A. Bauer	June 23, 1921	Steel III	8	0.48 ± 0.16 cm	-3.6×10^{-7}	1.01
H. A. Lorentz	Apr. 25, 1922	Armco iron	5	2.65 ± 0.45	4.3	1.20
C. A. Kotterman*	Apr. 26, 1922	Armco iron	16	2.32 ± 0.24	3.8	1.06
E. L. Nichols and E. Merritt	Apr. 26, 1922	Armco iron	8	2.37 ± 0.14	3.9	1.09
H. Crew	June 16, 1922	Steel III	8	2.8 ± 0.5	4.0	1.12
R. C. Tolman, S. Karrer, and E. W. Guernsey	Aug. 22, 1922	Cobalt II	10	3.6 ± 0.3	4.6	1.29
Mrs. C. A. Kotterman	Oct. 1, 1922	Steel III	8	3.75 ± 0.15	3.76	1.05
W. Duane, E. M. Terry, and W. E. Tisdale	Dec. 12, 1922	Permalloy	6	4.45 ± 0.12	3.75	1.05
W. S. Adams and C. G. Abbot	Dec. 15, 1922	Permalloy	10	4.72 ± 0.10	3.9	1.09
A. Sommerfeld and F. J. Bates	March 8, 1923	Permalloy	8	4.95 ± 0.05	3.75	1.05
A. P. Wills and Mr. and Mrs. S. L. Quimby	Apr. 20, 1923	Steel III	10	4.3 ± 0.1	3.96	1.11
P. S. Epstein	June 2, 1923	Steel III	8	1.57 ± 0.11	3.6	1.01
N. Bohr and P. D. Foote	Nov. 25, 1923	Cobalt II	8	2.58 ± 0.12	3.4	0.95

* Mr. Kotterman was our research assistant, but he had never before made the observations himself.

greater than when the elaborate precautions necessary for precise work could be taken; but the mean value of the gyromagnetic ratio differs by only three per cent from that of the precise work, and the mean error is only about six per cent.

The mean value of λ from all these observations is $-(3.87 \pm 0.22)\ 10^{-7}$ gauss per rev. per second, corresponding to $\rho = (1.08 \pm 0.06)\ m/e$. The mean value of λ from the precise (nocturnal) observations at the highest speed is -3.76×10^{-7} gauss per rev. per second, corresponding to $\rho = 1.05\ m/e$.

The mean values of $\rho e/m$ obtained for the four rotors in these observations are given in Table 41 together with the values obtained from the standard measurements made on the same rotors under the best conditions.

Table 40 is published not only because it shows what can be done under extremely adverse conditions, but also because a great deal of interest has been expressed in it by the many who have seen it.

§ 62. SLIGHT REVISION OF THE WASHINGTON RESULTS ON THE INTRINSIC MAGNETIC INTENSITY OF ROTATION $\lambda = 2\pi\rho$. The quantity $\rho\ (= \lambda/2\pi)$ derived from the observations in II is given by Equation (8), II, p. 170 on division by 2π. The equation for ρ may thus be written

$$\rho = A/Q$$

where Q is the ratio of *rotor sensitivity* to *magnetometer sensitivity*, and A contains the other observed quantities and constants. Of the adequate precision of the quantities involved in A there was never any doubt; but suspicion of transient defects in the insulation of one of the coils on which Q directly depends, aroused long after the main experiments were completed and the results calculated, made desirable certain additional measurements. The new tests consisted chiefly of (1) additional determinations of the quantity Q for Steel III (viz. Q_S), which will be denoted by Q_S', and (2) series of determinations for each of the other rotors of the ratio of Q to Q_S, which will be designated by R.

(1) It has been practicable to improve somewhat the calculations of the three values of Q_S' obtained by two independent methods and given in II § 23 (1), the new values being -0.8, 0.1 and -1.1 per cent greater than the value of Q_S for the same magnetometer. The mean is -0.6 ± 0.3 per cent greater, which would indicate (except for probable errors in magnetometer

TABLE 41

COMPARISON OF ROUGH AND STANDARD VALUES OF $\rho e/m$

Rotor	Steel III	Armco Iron	Cobalt II	Permalloy
No. sets	5	3	2	3
$\rho e/m$	1.06±0.04	1.12±0.06	1.14±0.017	1.06±0.02
$\rho e/m$ std.	1.05	1.03	1.07	1.06

settings, etc., for Q_S') that the values of λ and ρ for Steel III are *too small* by this amount.

(2) The check on the values of Q and ρ obtained in the main observations of II provided by the new measurements of R can be made more perspicuous by the arrangement of results given in Table 42 than by that of II, Table III, p. 168. The means of the values of R in II, Table III, are given in column 6. Column 5 gives the values Q_C of Q calculated for the different rotors from Q_S (for the two different magnetometer systems) and R by the formula $Q_C = RQ_S$. Column 4 gives the Washington values of Q (series B); column 7, the Washington values of ρ; and column 8, the values ρ_C of ρ calculated from the formula $\rho_C = \rho Q/Q_S$*; column 9, the quantities $\rho_C - \rho$. The calculation of ρ_C depends for its validity upon the assumption that the ratio of the moment of the upper and lower magnetometer systems remained sensibly unaltered throughout the series. In much of the earlier work we determined this ratio frequently and with considerable precision. But this became unnecessary in the later work on account of the more elaborate and more nearly exact procedure adopted for securing *effective* astaticism; and in the course of Series B we made only two rough measurements of the ratio for the 3-magnet system magnetometer, a few days apart, near the middle of the series. That the ratio of the moments remained constant, or approximately constant, is indicated by the fact that $\rho_C - \rho$ shows no systematic change with the time as the series proceeded.

The degree of agreement between columns 4 and 5 shows that in nearly every case no defect of insulation could be responsible for any consequential excess of the observed value of ρ above its true value.

An exception appears to occur in the case of the observations on Steel IV for September 30–October 2, for which Q is much smaller (3 per cent) than Q_C, ρ much greater (3 per cent) than ρ_C, while for the December observations the agreement is excellent. The value of Q for the earlier group of observations was obtained from two closely concordant determinations made on different days; nevertheless I am convinced that the leads of the calibrating coil must then have been in an exceptional position in which a slight short-circuit occurred during both tests.† The September–October value of Q has therefore been rejected, and the observations reduced with the value of Q obtained in December, which agrees well with Q_C, and brings ρ and ρ_C into good agreement.

While writing VI Table 23-1, I considered the evidence to indicate that the value of Q for the October observations on electrolytic iron was in error, as the earlier value for Steel IV seemed to be, and made a similar correction. The close and simultaneous agreement, however, between ρ and ρ_C and between Q and Q_C for this rotor shows that the correction was not justified. In spite of diligent searches, I have never succeeded in finding any explanation of the large value of ρ for this rotor. (See footnote p. 453.)

The values of ρ (and ρ_C) in the table differ slightly from those printed in VI Table 23-1, for three reasons. In the first place, the values of the end correction K obtained by a later and somewhat more direct process (see II § 22, footnote 30) have been applied in place of the earlier values. This increases the values of ρ (and ρ_C) by

* This follows at once from the relation $\rho_c = A/Q_c$ and $\rho = A/Q$. The values of ρ_c given in VI § 23 were correctly calculated from the above formula; but the ratio A_s/A, which should multiply ρ in Equation 18-1, was omitted in the text. When this ratio is inserted the formula becomes equivalent to that just given.

† The evidence now seems to indicate that a partial short-circuit occurred on one occasion before the beginning of Series A and Series B, when a rather rough resistance measurement *appeared* to give too low a value. This was rejected as in some way incorrect because it was quickly followed by a perfectly conclusive insulation measurement which showed no defect whatever. It now seems quite probable that both measurements were correct, the leads being arranged in an exceptional manner in one case.

TABLE 42

SLIGHTLY REVISED VALUES OF $\rho e/m$ FROM WASHINGTON INVESTIGATION ($e/m = 1.757 \times 10^7$ e.m.u.)

1 Rotor	2 Date	3 No. Sets	4 Q	5 Q_c	6 R	7 $\rho e/m$	8 $\rho_c e/m$	9 $(\rho_c - \rho)e/m$
Electrolytic iron II	Oct. 20–21	4	2.337	2.350	0.991±0.012	1.108	1.102	−0.006
	Oct. 23–26	9	(2.337)	2.350	0.991±0.012	1.094	1.088	−0.006
	Dec. 8	5	2.379	2.350	0.991±0.012	1.069	1.082	+0.013
	Means	18				1.091	1.090	
Armco iron	Oct. 14	10	1.684	1.691	0.713±0.002	1.027	1.023	−0.004
Norway iron	Dec. 29–30	6	1.885	1.849	0.780±0.005	1.032	1.052	+0.020
Steel I	Dec. 25–27	9	1.802	1.811	0.764±0.002	1.047	1.042	−0.005
Steel III	Sept. 10–12	6	2.355		1.000	1.044		
	Oct. 3–4	5	2.371		1.000	1.051		
	Dec. 3–Jan. 1	4	2.371		1.000	1.037		
	Means	15				1.044		
Steel IV	*Sept. 30–Oct. 2	8	(2.026)*	2.037	0.859±0.003	1.073	1.067	−0.006
	Dec. 22–24	13	2.026	2.037	0.859±0.003	1.043	1.038	−0.005
	Means	21				1.054	1.049	
Nickel I	Nov. 8–10	9	1.752	1.802	0.760±0.002	1.049	1.020	−0.029
Nickel III	Nov. 2–3	4	2.199	2.224	0.938±0.002	1.025	1.014	−0.011
	Nov. 4–5	6	(2.199)	2.224	0.938±0.002	1.006	0.994	−0.012
	Means	10				1.014	1.002	
Cobalt II	Nov. 19–21	6	1.937	1.878	0.792±0.004	1.077	1.111	+0.034
	Nov. 22–27	7	1.943	1.878	0.792±0.004	1.066	1.103	+0.037
	Means	13				1.071	1.107	
Heusler alloy I	Dec. 9–12	10	1.145	1.124	0.474±0.001	1.012	1.031	+0.019
Permalloy	Aug. 31–Sept. 1	4	2.960	2.946	1.251±0.005	1.049	1.054	+0.005
	Sept. 2–6	7	1.386AF		(1.255 AF)	1.074		
	Dec. 13–14	5	2.936	2.966	(1.251±0.005)	1.034	1.023	−0.011
	Means	16				1.055†	1.037‡	
Nickel-iron	Nov. 16–18	10	1.047	1.046	0.441±0.002	1.017	1.018	+0.001
Cobalt-iron	Dec. 16–18	6	2.585	2.610	1.101±0.004	1.068	1.057	−0.011
Cobalt-nickel	Oct. 5–6	4	1.651	1.641	0.692±0.002	1.069	1.076	+0.007

* The uncorrected values of Q and $\rho e/m$ are, respectively, $Q = 1.973$ and $\rho e/m = 1.102$.

† For all 16 sets. ‡ For the 9 sets with equatorial position of rotor.

amounts varying from 0.0 per cent to 0.3 per cent—negligible or almost negligible quantities when the other errors are considered. In the second place, the values of ρ and ρ_C are given separately for all the groups of II Table XIII; while, for brevity, only mean values were given in VI. In the third place, the small corrections discussed below in § 63, and due to the motion in azimuth of the lower magnetometer magnet, have been applied.

§ 63. Correction Due to the Motion in Azimuth of the Magnetometer System. In II § 40 the possible error due to the change of magnetization of the rotor arising from the motion in azimuth of the lower magnetometer magnet is considered. For the magnetometer with the six-magnet systems in the principal position LEQ the change in the axial intensity *at the center of the rotor* produced in this way, is, for the Cobalt-Iron rotor, about 1½ per cent* of that produced by reversing the rotation. The *mean* axial intensity along the length of the rotor produced in this way is of course much smaller, but its effect is impossible to compute. No error arises from this source if the calibrations are made with intensities equal to the intrinsic intensity of rotation. But in most of the calibrations the intensity was some sixty times as great as the maximum intensity of rotation. Thus, while the effect practically vanishes in these calibrations, it may remain in the rotation experiments.

On this account, although the maximum possible effect is small, it has seemed desirable to make an experimental determination of the actual error involved, as follows.

To determine the correction for the magnetometer position LEQ a standard solenoid C, with dimensions nearly the same as those of the short standard used in the Washington experiments, was placed with its axis horizontal, as in Fig. 12. Two small short coils A and B wound on the same bobbin, and nearly alike, were mounted with their axes horizontal and 12.9 cm. from the axis of C and opposite its center. A was connected in series with another coil A' and, through a reversing switch, with a battery and rheostat. A could be replaced at will with the coil C. B was connected in series with another coil B', and with a ballistic galvanometer and rheostat. By moving A' relatively to B' the mutual inductance of AA' and BB' was made nearly to vanish, as is usual with induction balances.

Then a current 0.06 ampere was repeatedly reversed through coils AA' (1) when the cobalt-iron rotor was mounted symmetrically in coil C and (2) when it was removed to a distance. The mean throw in the first case was 0.467 cm.; that in the second, 0.365 cm., difference 0.102 cm. The area of the coil A was 13.8×10^3 cm², so that its magnetic moment was $13.8 \times 10^3 \times 6.0 \times 10^{-3} = 83$ e.m.u. Thus the change in the magneti-

* Instead of 3 per cent, given in II, through the omission of a factor 2.

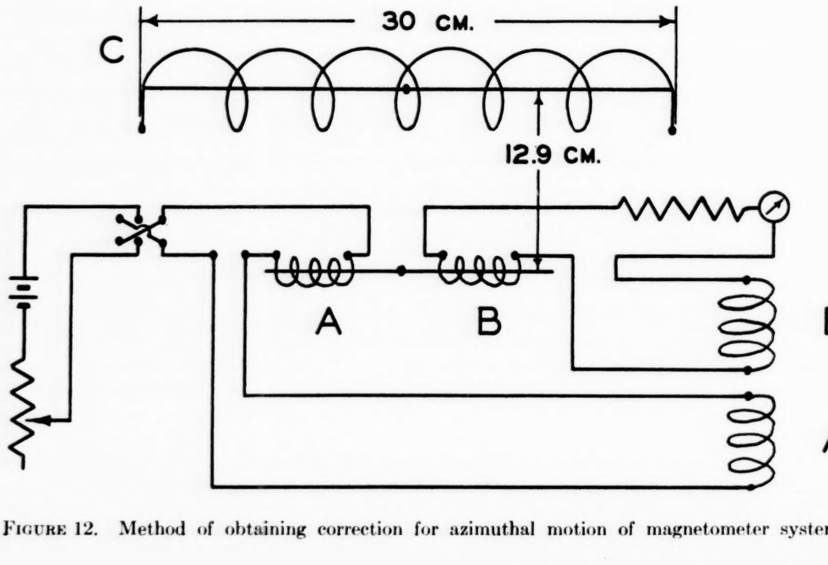

Figure 12. Method of obtaining correction for azimuthal motion of magnetometer system.

zation of the cobalt-iron rotor due to the reversal of the moment 83 e.m.u. of coil A with its axis 12.9 cm. from that of the rotor produced a deflection of 0.102 cm. through induction with A', similarly situated with A as regards the rotor.

The moment of a 6-magnet system of the magnetometer used in the Washington experiments was about 0.4 e.m.u.; and the magnet, normally at right angles to the axis of the rotor and with 12.9 cm. between their centers, was turned through an angle of about 0.0031 radian when the rotation of the cobalt-iron rotor at 61 r.p.s. was reversed. Thus if this magnet were to replace coil A in the above arangement and to be turned through the angle mentioned, it would produce by induction with A', a deflection $(0.2 \times 3.1 \times 10^{-3})/83 \cdot 0.102$ cm. $= 7.6 \times 10^{-7}$ cm.

Coils A and A' were now replaced by coil C, and the mean throw was obtained for a number of reversals of a current 0.006 ampere in C. The mean throw was about 0.71 cm. Had the coil C been very long, the intensity it impressed on the rotor would have been, in gauss/ampere, 21.7 (its constant if infinitely long) $\times$ 0.006 = 0.13 gauss, and the deflection due to the *change of magnetization of the rotor alone* would have been $0.71 \times (1-0.006)$ cm., the quantity 0.006 here being the correction factor K for the Cobalt-Iron rotor (see II pp. 155–165). Thus the reversal of an intensity 0.13 gauss acting on the rotor would produce a throw of about 0.7 cm.

The intrinsic magnetic intensity of rotation at 61 r.p.s., is about 23×10^{-6} gauss. The reversal of this intensity in the last described arrangement would produce a throw $(23 \times 10^{-6}/0.13) \times 0.7 = 124 \times 10^{-6}$ cm.

Thus in the actual rotation experiment with the Cobalt-Iron rotor, the effect of the motion of the 6-magnet magnetometer magnet would be to make the observed effect too large by the fraction $7.6 \times 10^{-7}/124 \times 10^{-6} = 6.1 \times 10^{-3}$ or about 1 part in 160.

The fraction is nearly proportional to the specific moment of the rotor (II § 14, Table I) and to the absolute or magnetometer sensitivity A (II, Table XIII, column 28), as well as to the moment of the magnetometer magnet. It is thus one half as great for the 3-magnet system as for the 6-magnet system. The fraction ranges from about 0.1 per cent for the Heusler alloy and Hopkinson alloy rotors used with the 3-magnet systems to about 0.7 per cent for the Permalloy rotor when used with the 6-magnet systems.

Experiments similar to those just described, but with the Permalloy rotor, and with the small test coils placed with reference to it in the position designated in II as AF (about 17 + 15 cm. from the center of the rotor and on its axis produced), showed that in this arrangement the correction was inappreciable.

Part IV.

COMPARISON OF RESULTS OBTAINED BY THE TWO GYROMAGNETIC EFFECTS

§ 64. **The Washington and California Results: Summary and Comparison.** In Table 43 are collected the mean values of $\rho e/m$ obtained in this investigation by means of the Einstein and de Haas effect, and also the mean values obtained by means of the Barnett effect in the Washington observations, taken from Table 42. The discrepancies are all small except in the case of Yensen's Electrolytic iron and Cobalt-Iron. For these discrepancies of several per cent, no explanation has been found.*

The mean values of $\rho e/m$ obtained from the two investigations for materials studied in both are in close agreement, 1.053 from the Barnett effect, and 1.047 from the Einstein and de Haas effect.

The values of $\rho e/m$ in Table 42 are from the best observations in II, viz., the 159 sets at the highest speed (ca. 61 r.p.s.) in series B of that paper. The mean for all the 14 rotors is 1.046. The means for all the four speeds used agreed well, as shown in II and VI. The next best group in the series is that for the next lower speed, ca. 31 r.p.s., which gave as the mean, from 163 sets, 1.055. Moreover, the preceding series, Series A, gave for the higher speed, from 150 sets, the value 1.054; and for the lower speed, from 100 sets, the value 1.060. The degree of agreement between these results and the mean for the new investigation on the converse effect much more than justifies the authors of II in their claim of an error for their mean not greater than 2 per cent. Indeed it now appears that for *no one of the materials* used in II, with the exception of Electrolytic iron and Cobalt-Iron, is the mean value of ρ from II in error by more than this amount; and the error is usually much less. The values of ρ for Permalloy and Iron, as determined

* Measurements made in a new investigation on magnetization by rotation since this was written do not confirm the high Washington values for these substances, while they do give high values for Cobalt and Cobalt-Nickel. The work is not yet completed.

TABLE 43

$\rho e/m$ from Barnett Effect and Einstein and de Haas Effect

Material	Barnett	E. & de H.	Material	Barnett	E. & deH.
Armco iron	1.027	1.032	Hopkinson's Iron-Nickel alloy	1.017	1.024
Norway iron	1.032	——	Permalloy	1.055	1.047
Honda iron	——	1.023	Cobalt	1.071	1.090
Yensen iron (electrolytic)	1.091	1.034	Cu-Cobalt	——	1.082
Cold-rolled steel	1.048	1.039	Cobalt-Iron	1.068	1.027
Nickel	1.032	1.052	Cobalt-Nickel	1.069	1.077
Hipernik	——	1.052	Heusler alloy	1.012	——*

Mean $\rho e/m$ from Barnett Effect = 1.053 for materials studied in *both* investigations.
Mean $\rho e/m$ from Einstein and de Haas Effect = 1.047 for materials studied in *both* investigations.

* The value given in § 54 is not inserted here because of its preliminary character.

TABLE 44

Historical Tabulation of $\rho e/m$ for Ferromagnetic Substances

Date	Investigators	Effect Investigated	Material	$\rho e/m$	Sign of Charge in Ampèrian Whirl
1914	Barnett	Barnett	Steel	1.01	negative
1915	Barnett	Barnett	Steel	0.95	negative
1915	Einstein & de Haas	E. & de H.	Iron	2.0	indeterminate
1916	Einstein & de Haas	E. & de H.	Iron	—	negative
1917	Barnett	Barnett	Iron, Nickel, & Cobalt	>1<2	negative
1918	J. Q. Stewart	E. & de H.	Iron	1.02	negative
			Nickel	0.94	negative
1919	E. Beck	E. & de H.	Iron	1.06	negative
			Nickel	1.14	negative
1919	G. Arvidsson	E. & de H.	Iron	0.94	———
1922	H. Claassen*	E. & de H.	Soft iron to Glass-hard steel	1.13 to 2.4	———
1923	Chattock & Bates	E. & de H.	Iron	1.01	negative
			Nickel	1.01	negative
1920–25	Barnett & Barnett	Barnett	Numerous†	1.01 to 1.09	negative
1923	Sucksmith & Bates	E. & de H.	Fe, Ni, Heusler alloy‡	0.99+to1.00−	negative
1925	Sucksmith	E. & de H.	Cobalt‡	1.02	negative
			Magnetite	0.98	negative
1934	Sucksmith	E. & de H.	Cu-Ni alloy‡	1.05	———
1933–35	Coeterier & Scherrer	E. & de H.	Iron	1.00+	negative
			Pryrhotite	3.2	negative
1935	Raychaudhuri	E. & de H.	Iron Oxides	1.03 & 1.02	———
1931–38	Barnett	E. & de H.	Numerous¶	1.02 to 1.09	negative
1938	Kikoin & Goobar	E. & de H.	Iron**	1.03–	———

* See abstract of Hamburg dissertation in Phys. Ber. **4**: p. 359, 1923. I am indebted to Prof. R. Becker for this reference.

† See Tables 42 and 43.

‡ See Table 38.

¶ See Table 43 and the numerous earlier tables in this work.

§ As pointed out by H. A. Lorentz. See A. Einstein, Verh. d. D. Phys. Ges. **18**: 173, 1916.

** See § 56, above, at the end.

from the present investigation, are probably correct to one-half per cent; while it is unlikely that the mean value of ρ given in Table 43 from this investigation for any of the other materials is in error by much more than one per cent.

§ 65. SUMMARY OF ALL RESULTS HITHERTO OBTAINED ON GYROMAGNETIC RATIOS FOR FERROMAGNETIC SUBSTANCES. References to gyromagnetic effects in the literature of physics of the last twenty years and more, including even very recent literature, contain so many errors as to the magnitudes of the effects and the history of their discovery and measurement that it seems important, in concluding this paper, to present the adjoining historical table (Table 44) in which the chief results of all the investigations are given, and in chronological order.

§ 66. ACKNOWLEDGMENTS. In the course of this work a great deal of help has been received from many individuals and organizations, and it is a pleasure to express here my gratitude to them all. Some of them are mentioned in the preceding text. Special mention should be made of those who have acted, *seriatim*, as laboratory assistants since the completion of V, viz. Mr. B. W. Sorge, Mr. K. K. Illingsworth, Mr. L. L. Young and especially Dr. S. J. Broadwell. The work has been long continued and expensive, and could not have been accomplished without the joint support of the Board of Research of the University of California, the Norman Bridge Laboratory of the California Institute, in which the experimental work has been done, and the Carnegie Institution of Washington, to which I continue indebted for the use of most of the fixed equipment.

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